Solving Equations in General:
Solving equations is very common in algebra. The goal is to have a variable, by itself, equal to a number. The goal is achieved by repeatedly modifying both sides and then simplifying both sides.

This modify/simplify process is used on different types of equations. Some common types are shown in the following table. Notice that equations are named for the variable status.

Name of Equation	Variable Status	Examples
Linear	Variable has an exponent of one.	$2x = 4x - 3$
Radical	Variable is under a radical.	$\sqrt{3x} = 3x + 5$
Quadratic	The greatest exponent on any variable is two.	$3x^2 = 4x - 3$

Quadratic Equation
The general form of a quadratic equation is,

$ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

Examples:

- $3x^2 = 4x - 3$
- $3x^2 + 10x - 4 = 0$
- $x^2 = 0$

There are different methods to solve quadratic equations. The most basic method uses the square root property.

Square Root Property

If $x^2 = k$, then $x = +\sqrt{k}$ or $x = -\sqrt{k}$.

Example: If $x^2 = 9$, then $x = +\sqrt{9}$ or $x = -\sqrt{9}$; and this simplifies to $x = 3$ or $x = -3$.

We can check the equation with both values.

Check $x^2 = 9$ for $x = 3$

$$(3)^2 \stackrel{?}{=} 9$$
$$(3)(3) \stackrel{?}{=} 9$$
$$9 \stackrel{\checkmark}{=} 9$$

Check $x^2 = 9$ for $x = -3$

$$(-3)^2 \stackrel{?}{=} 9$$
$$(-3)(-3) \stackrel{?}{=} 9$$
$$9 \stackrel{\checkmark}{=} 9$$

The property is usually abbreviated as:

If $x^2 = k$, then $x = \pm\sqrt{k}$. Note, the $\pm$ symbol means plus or minus.

Steps to Solve a Quadratic Equation Using the Square Root Property

1. Write original problem.
2. **Note**: This method is only used when there is just one variable squared or one variable with a constant term squared, such as, $3x^2 = 10$ or $(2x - 7)^2 = 10$.
3. Isolate the variable square or the parenthesis square on left side of equation.
   a. First, add or subtract terms on both sides.
   b. Secondly, divide both sides by number and sign in front of variable or parenthesis.
4. Take square root of each side and make sure right side has a $\pm$ before the square root sign.
5. Simplify both sides.
   a. The left side will just be the radicand without the power of 2.
   b. On right side, simplify square root.
6. Continue to solve the equation that now has no variable under a square root.
   a. If a term has to be added or subtracted on each side, make sure that on right side it placed in front of the $\pm$ sign.
   b. If each side has to be divided by a coefficient, keep the $\pm$ on right side.
   c. If there is no square root, split equation into two equations at the $\pm$ and simplify.
7. Check if necessary.
8. Write the solution in set form.

Example 1: Solve $3(2x + 4)^2 - 8 = 10$

Comments	**Steps**
Write original problem.	$3(2x + 4)^2 - 8 = 10$
Modify each side with the + 8.	$3(2x + 4)^2 - 8 + 8 = 10 + 8$
Simplify each side.	$3(2x + 4)^2 = 18$
Modify each side by dividing by 3 on each side.	$\frac{3(2x+4)^2}{3} = \frac{18}{3}$
Simplify each side.	$(2x + 4)^2 = 6$
Modify each side with a square root. **Remember the $\pm$ on the right side.**	$\sqrt{(2x + 4)^2} = \pm \sqrt{6}$
Simplify each side.	$2x + 4 = \pm \sqrt{6}$
Modify each side with a $-$ 4. Note the $-$ 4 before the $\pm$ on right side.	$2x + 4 - 4 = - 4 \pm \sqrt{6}$
Simplify each side.	$2x = - 4 \pm \sqrt{6}$
Modify by dividing by 2 on each side. **Note: The entire right side is divided by 2.**	$\frac{2x}{2} = \frac{-4 \pm \sqrt{6}}{2}$
Simplify each side.	$x = \frac{-4 \pm \sqrt{6}}{2}$
Write solution in set form.	Solution: $\left\{\frac{-4 \pm \sqrt{6}}{2}\right\}$

Example 2: Solve $2x^2 = 24$

Comments	**Steps**
Write original problem.	$2x^2 = 24$
Modify a division by 2 on each side.	$\frac{2x^2}{2} = \frac{24}{2}$
Simplify each side.	$x^2 = 12$

Modify each side with a square root. **Remember the $\pm$ on the right side.**	$\sqrt{x^2} = \pm\sqrt{12}$
Simplify each side. Notice the $\sqrt{12}$ is simplified on the side.	$x = \pm\ 2\sqrt{3}$
Put answer in set form.	Solution: $\{\pm\ 2\sqrt{3}\}$

$$\sqrt{12} = \sqrt{4 \cdot 3}$$
$$= \sqrt{4} \cdot \sqrt{3}$$
$$= 2\sqrt{3}$$

Example 3: Solve $2(x + 4)^2 = 18$

Comments	**Steps**
Write original problem.	$2(x + 4)^2 = 18$
Modify a division by 2 on each side.	$\frac{2(x+4)^2}{2} = \frac{18}{2}$
Simplify each side.	$(x + 4)^2 = 9$
Modify each side with a square root. **Remember the $\pm$ on the right side.**	$\sqrt{(x + 4)^2} = \sqrt{9}$
Simplify each side.	$x + 4 = \pm 3$
Modify each side with a $- 4$. Note the $- 4$ before the $\pm$ on right side.	$x + 4 - 4 = - 4 \pm 3$
Simplify each side.	$x = - 4 \pm 3$
Split equation into two equations at the $\pm$ because there is no square root.	$x = - 4 + 3$ or $x = - 4 - 3$
Simplify both equations.	$x = - 1$ or $x = - 7$
Put solution in set form.	Solution: $\{- 1, - 7\}$