Steps to Solve a Quadratic Equation Where Complex Solution May Be Used

1. W.O.P
2. The square root of a negative number should be replaced by i and the square root of a positive number. For example, $\sqrt{-23}$ becomes $i\sqrt{23}$.
3. Continue to simplify square root and fraction from previous steps.

Example: Solve $7x^2 - 2x + 4 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; a = 7, \quad b = -2, \quad c = 4$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(4)}}{2(7)}$$

$$x = \frac{2 \pm \sqrt{4 - 28(4)}}{14}$$

$$x = \frac{2 \pm \sqrt{4 - 112}}{14}$$

$$x = \frac{2 \pm \sqrt{-108}}{14}$$

$$x = \frac{2 \pm i\sqrt{108}}{14}$$

$$x = \frac{2 \pm i(6)\sqrt{3}}{14}$$

$$x = \frac{2 \pm 6i\sqrt{3}}{14}$$

$$x = \frac{2}{14} + \frac{6i\sqrt{3}}{14} \text{ or } x = \frac{2}{14} + \frac{6i\sqrt{3}}{14}$$

$$x = \frac{1}{7} + \frac{3i\sqrt{3}}{7} \text{ or } x = \frac{1}{7} - \frac{3i\sqrt{3}}{7}$$

Notice the $\sqrt{-108}$ is changed to an $i\sqrt{108}$.

Side Work

$$\sqrt{108} = \sqrt{(36)(3)}$$
$$= \sqrt{36}\sqrt{3}$$
$$= 6\sqrt{3}$$

Solution: $\left\{\frac{1}{7} + \frac{3i\sqrt{3}}{7}, \frac{1}{7} - \frac{3i\sqrt{3}}{7}\right\}$