Steps to Simplify a Rational Expression

1. Write original problem.

2. Prep each fraction by doing the following:
   a. Reorder the terms in the numerator or denominator if they are not in standard form. Standard form is highest to lowest degree.
   b. Factor out a negative one if the numerator or denominator if leading term is not positive. Keep the negative one in a parenthesis as well as each polynomial in the numerator or denominator.
   c. Simplify signs if a negative one was factored in numerator and/or denominator. Please look at the three possible cases:

   Case 1: There is negative one in numerator and denominator. This is simplified by dividing a negative by a negative and the result is a positive fraction.

$$\frac{(-1)(x+4)}{(-1)(x-2)} = \frac{(x+4)}{(x-2)}$$

   Case 2: There is negative one just in numerator. This is simplified by dividing a negative by a positive and the result is a negative fraction.

$$\frac{(-1)(x+4)}{(x-2)} = -\frac{(x+4)}{(x-2)}$$

   Case 3: There is negative one just in denominator. This is simplified by dividing a positive by a negative and the result is a negative fraction.

$$\frac{(x+4)}{(-1)(x-2)} = -\frac{(x+4)}{(x-2)}$$

3. Factor numerator and denominator completely. Keep all factors in parenthesis. Use the side of problem for multi-step factoring. For monomial factors, factor the coefficient and variables completely. For example:

$$\frac{x^2-4}{6x^3} = \frac{(x+2)(x-2)}{(2)(3)(x)(x)(x)}$$

4. Cancel like factors in numerator and denominator.

5. If restrictions are required, we have to keep track of any canceled factor that contains a variable. Please make an area on side of paper with the heading: "**Restrictions?**" Copy the canceled factors into the **Restrictions** area.

6. Write remaining factors.

7. Do not multiply remaining factors unless they are monomial factors. For example:

$$\frac{(x+2)(x-2)}{(2)(3)(x)(x)} = \frac{(x+2)(x-2)}{6x^2}$$

8. If only one factor remains in numerator or denominator, the parenthesis are not needed. For example:

$$\frac{(x-6)}{(x+5)} = \frac{x-6}{x+5}$$

9. If restrictions are requested, look at the factors in the **Restricted?** area. For any factor that is **no longer in the denominator**, you set it not equal to zero and solve. The value will be put by last step of the problem. More explanation of why we do this is shown in the first example.

Example 1: Simplify: $\dfrac{x^2-3x-10}{x^2-6x+5}$ and write restricted domain.

Notes:	Problem
Write original problem; notice numerator and denominator are in standard form and the leading terms are positive. No ***prep*** is needed.	$\dfrac{x^2-3x-10}{x^2-6x+5} = \dfrac{\cancel{(x-5)}(x+2)}{\cancel{(x-5)}(x-1)}$
Factor numerator and denominator and cancel like factors.	
The canceled factor $(x-5)$ contains a variable so we make an area for possible **Restrictions?** The factor is copied into the area.	**Restrictions?** $(x-5)$
Write remaining factors.	$= \dfrac{(x+2)}{(x-1)}$
	$(x-5) \neq 0$ $x-5 \neq 0$ $x-5+5 \neq 0+5$ $x \neq 5$
Parenthesis are cleared at final step because they are not needed.	$= \dfrac{x+2}{x-1};\ x \neq 5$
We notice that there is not another $(x-5)$ in the denominator so we need to find out when this factor goes to zero.	
Put restriction by the last step. More explanation follows this example.	

In the above example a restriction is added $x \neq 5$. The reason for this is that in the original fraction,

NOTE: All of the white space is not needed on the main part of problem. It was added to show explanation of the

$\frac{x^2 - 3x - 10}{x^2 - 6x + 5}$, a value of $x = 5$ would cause the denominator to go to zero and thus the evaluated fraction would be undefined. See the following:

Evaluate $\frac{x^2 - 3x - 10}{x^2 - 6x + 5}$ for $x = 5$

$$= \frac{(5)^2 - 3(5) - 10}{(5)^2 - 6(5) + 5}$$

$$= \frac{25 - 15 - 10}{25 - 30 + 5}$$

$$= \frac{10 - 10}{-5 + 5}$$

$$= \frac{0}{0}$$ **undefined**

If we evaluate the final expression, $\frac{x+2}{x-1}$, for $x = 5$, the evaluated expression is not undefined.

The value, $x = 5$, did cause the original expression to be undefined and that is why $x \neq 5$ has to be put by the final simplified expression. If a problem has a restriction at the start, and we equivalent expressions on each step, the final simplified form also has to have the same restriction from the start.

Example 2: Simplify**:** $\frac{25 - x^2}{x^2 - 10x + 25}$ and write restricted domain.

Notes:	Problem
Write original problem and notice numerator is not in standard form. We have to prep the numerator. We will do all work on the side so that we do not have to keep writing the faction. There a negative numerator divided by a positive denominator and fraction	$\frac{25 - x^2}{x^2 - 10x + 25} = \frac{(-1)(x^2 - 25)}{(x^2 - 10x + 25)}$ The terms are reordered. The leading sign is not positive and thus a negative one is factored out. $25 - x^2 = -x^2 + 25$ $= (-1)(x^2 - 25)$

Notes:	Problem
becomes negative.	$= -\dfrac{(x^2 - 25)}{(x^2 - 10x + 25)}$
Factor the numerator and denominator and cancel like factors.	$= -\dfrac{(x+5)\cancel{(x-5)}}{(x-5)\cancel{(x-5)}}$ **Restrictions?** $(x-5)$
The canceled factor has a variable so it is put in the area for **Restrictions?**	
Write remaining factors.	$= -\dfrac{(x+5)}{(x-5)}$
Parenthesis can be cleared.	$= -\dfrac{x+5}{x-5}$
The $(x-5)$ factor was canceled but there is another copy of $(x-5)$ in the denominator. No restriction has to be added.	

The reason we do not have to find out when $(x-5)$ has a value of zero and insert it on the last simplified step is that in the beginning of the problem a value of $x = 5$ would cause the denominator to go to zero and the fraction would be undefined. In the last step a value of $x = 5$ would still cause the simplified form to have zero in the denominator and the fraction would still be undefined. We do not have to add any more information.