Graphing Quadratic Functions

Any equation of the form, $y = ax^2 + bx + c$ or in function form of $f(x) = ax^2 + bx + c$ is called a quadratic function. The graph of a quadratic function is a parabola. The graph is made by using key features of the function.

Steps To Graph a Parabola

1. Write out original problem.

2. Put equation in standard form of:
   $y = ax^2 + bx + c$.

3. Find the following key features.
   a. **Open Direction**: If $a > 0$, graph opens up, if $a < 0$, graph opens down.
   b. **Vertex**: The ***x***-*coordinate* is: $x = \dfrac{-b}{2a}$; $a =$ __ , $b =$ __ . Substitute and solve for ***x***. The ***y***-*coordinate* is found by substituting in the ***x***-coordinate into the equation from step 2 and solve for ***y*.**

   c. **Axis of Symmetry**: The axis of symmetry is given by the equation, ***x*** = __ , which is the ***x***-*coordinate* of the vertex.

   d. ***y-intercept*:** by writing, Write equation from step 2, set up Let ***x*** = 0, substitute and solve for ***y***. Write "***y***-intercept is (0, …)".

   e. ***x-intercept***: Write equation from step 2, set up Let ***y*** = 0 and solve quadratic equation with factoring, or quadratic formula. **NOTE: Not all parabolas will have an *x* intercept.**

4. Set up a table to help organize the key features.

Constants	***a*** **= __ ,** ***b*** **= __ ,** ***c*** **= __**
Open Direction	*up* or *down*
Vertex	(___ , ___)
Axis of symmetry	***x*** = ____
y-*intercept*	(0, __)
x-*intercept*	(__ , 0) and (__ , 0). Note there may be just one point or no points.

5. Make a coordinate system with axis, tick marks, etc. Plot the vertex, draw a dashed line for the line of symmetry, and plot ***y-intercept*** point. Plot the ***x-intercept*** point or points if they exist.

6. If there are not at least two points on either side of the vertex, you will need more points to make an accurate parabola. Look for symmetrical points or choose your own values of ***x*** on one side of the parabola and then use equation from step 2 to find the corresponding ***y*** coordinates. Plot these new points.

7. Connect the points to make a parabola.

Example: Graph $y = x^2 - 4x + 3$.

Open Direction: Opens up because ***a* > *0***.

Vertex:

x-coordinate

$$x = \frac{-b}{2a};\ a = 1,\ \ b = -4$$
$$= \frac{-(-4)}{2(1)}$$
$$= \frac{4}{2}$$
$$= 2$$

y-coordinate

$$y = x^2 - 4x + 3;\ x = 2$$
$$= (2)^2 - 4(2) + 3$$
$$= 4 - 8 + 3$$
$$= -4 + 3$$
$$= -1$$

Intercepts:

y-intercept

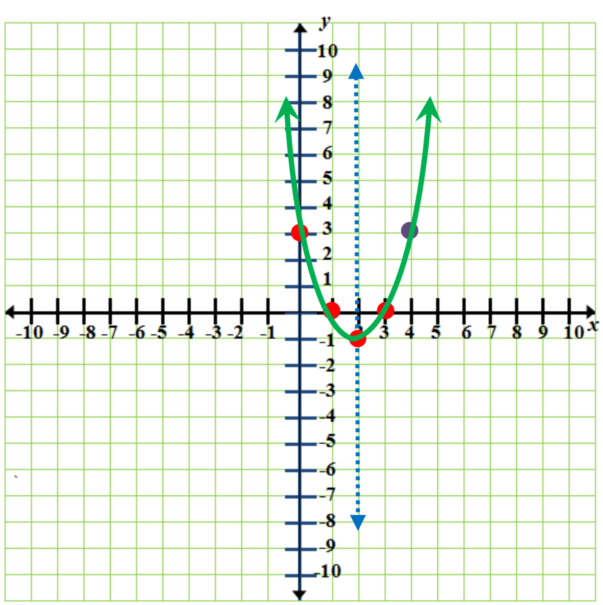
$$y = x^2 - 4x + 3;\ x = 0$$
$$= (0)^2 - 4(0) + 3$$
$$= 0 - 0 + 3$$
$$= 3$$

x-intercept

$$y = x^2 - 4x + 3;\ y = 0$$
$$(0) = x^2 - 4x + 3$$
$$0 = x^2 - 4x + 3$$
$$x^2 - 4x + 3 = 0$$

See last page for steps to find the points of the x-intercept.

Constants	$a = 1,\ b = -4,\ \ c = 3$
Open Direction	*up*
Vertex	$(2, -1)$
Axis of symmetry	$x = 2$
y-*intercept*	$(0, 3)$
x-*intercept*	$(3, 0)$ and $(1, 0)$



- We will now plot the points: $(2, -1)$, $(0, 3)$, $(3, 0)$ and $(1, 0)$ with **red** dots.
- We will make the axis of symmetry with a **blue** dashed line.
- We notice there is only one point on the right of the vertex so we will make a symmetrical point at $(4, 3)$ with a **purple** dot. The symmetrical point is made from point, $(0, 3)$.
- We will now connect the points with a parabola using a **green** curve.

Specific steps for finding the points for the x-intercept using the quadratic formula

$$Evaluate \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad for \quad a = 1, b = -4, c = 3$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$= \frac{4 \pm \sqrt{4}}{2}$$

$$= \frac{4 \pm 2}{2}$$

$$= \frac{4 \pm 2}{2}$$

$$= \frac{4 + 2}{2} or \frac{4 - 2}{2}$$

$$= \frac{6}{2} or \frac{2}{2}$$

$$x = 3, x = 2$$