Steps to Convert a Quadratic Function in Standard Form to Vertex Form by Completing the Square

1. Write original problem.
2. Put terms in order so that the function looks like:

   $$f(x) = ax^2 + bx + c$$

3. Always factor out the value of a. Note: Fractions may be used.
4. On side of problem compute the value of a **new term**. The term is the square of one-half of the coefficient of the x term.
5. Insert this **new term** and its **opposite** right after the x term.
6. Apply the distributive property and regroup the three terms in the ( ) and put the last two terms in another ( ). Put a is in front of both pairs of parenthesis.
7. Factor the terms in 1st ( ) by using the perfect square trinomial method. Simplify inside 2nd ( ) and simplify multiplication of a times the 2nd ( ).
8. Rewrite the factors of the trinomial as a $(\quad)^2$ so that the final form looks like:

   $f(x) = a(x-h)^2 + k$, with a vertex of (h, k).

Example: Convert $f(x) = 3x^2 + 2x + 5$ to Vertex Form.

$$f(x) = 3x^2 + 2x + 5$$

$$f(x) = 3\left(x^2 + \frac{2}{3}x + \frac{5}{3}\right)$$

$$f(x) = 3\left(x^2 + \frac{2}{3}x + \frac{1}{9} - \frac{1}{9} + \frac{5}{3}\right)$$

$$f(x) = 3\left(x^2 + \frac{2}{3}x + \frac{1}{9}\right) + 3\left(-\frac{1}{9} + \frac{5}{3}\right)$$

$$f(x) = 3\left(x + \frac{1}{3}\right)\left(x + \frac{1}{3}\right) + 3\left(-\frac{1}{9} + \frac{5(3)}{3(3)}\right)$$

$$f(x) = 3\left(x + \frac{1}{3}\right)^2 + 3\left(-\frac{1}{9} + \frac{15}{9}\right)$$

$$f(x) = 3\left(x + \frac{1}{3}\right)^2 + 3\left(\frac{14}{9}\right)$$

$$f(x) = 3\left(x + \frac{1}{3}\right)^2 + \frac{\cancel{3}}{1}\left(\frac{14}{(\cancel{3})(3)}\right)$$

$$f(x) = 3\left(x + \frac{1}{3}\right)^2 + \frac{14}{3}$$

Vertex will be $\left(-\frac{1}{3}, \frac{14}{3}\right)$.

$$\left[\frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\right)\right]^2 = \left[\frac{1}{3}\right]^2 = \frac{1}{9}$$

Computer new term on side of problem.

Insert this new term and its opposite back in problem.