

## Steps to do Long Division of Polynomials

1. Write out original problem.
2. Set up problem in long division format and make sure that that **all terms** are listed from the highest exponent down to a constant. Add terms if necessary and put each polynomial in standard form. For example, if the original dividend is:

$$2 + 4x^3 + 3x$$

The polynomial is not in standard form and is also missing the  $x^2$  term. The polynomial has to be written as:

$$4x^3 + 0x^2 + 3x + 2$$

3. Divide the first term in the divisor into the first term of the dividend and put the result up in the quotient area.
4. Multiply the term, you just put up in the divisor area, times each term in the divisor and put the result below the dividend.
5. Subtract the polynomial, you just placed by using the following process:
  - a. Put the opposite sign over each sign and use a different color.
  - b. Combine the terms in each column and you will now have a new dividend.
6. Bring down the rest of the old dividend.
7. Repeat **steps 3 – 6** until the new dividend has a degree less than the divisor.
8. If there is a remainder, the final answer is the polynomial you have in the quotient area plus the remainder over the divisor.

Note: The process of long division follows the acronym of **D, M, S, B**, that is, divide, multiply, subtract and bring down. The letters **D, M, S, B**, can be remembered by thinking of a family, **D** for Dad, **M** for Mom, **S** for Sister and **B** for Brother.

Example with notes.

|   |   |
|---|---|
| $(2 + 4x^3 + 3x) \div (x - 3)$ <p style="text-align: center;"><i>Quotient Area</i></p> $x - 3 \overline{)4x^3 + 0x^2 + 3x + 2}$ <p style="text-align: center;"> <span style="color: red;">↑</span>                      <span style="color: red;">↙ ↘</span><br/> <i>Divisor</i>              <i>Dividend</i> </p> $x - 3 \overline{)4x^3 + 0x^2 + 3x + 2}$ <p style="text-align: center;"><math>4x^2 \leftarrow</math></p> $x - 3 \overline{)4x^3 + 0x^2 + 3x + 2}$ <p style="text-align: center;"><math>4x^3 - 12x^2 \leftarrow</math></p> $x - 3 \overline{)4x^3 + 0x^2 + 3x + 2}$ <p style="text-align: center;"><math>-4x^3 + 12x^2</math></p> $x - 3 \overline{)4x^3 + 0x^2 + 3x + 2}$ <p style="text-align: center;"><math>-4x^3 + 12x^2</math></p> <p style="text-align: center;"><math>12x^2 \leftarrow</math></p> $x - 3 \overline{)4x^3 + 0x^2 + 3x + 2}$ <p style="text-align: center;"><math>-4x^3 + 12x^2 \downarrow</math></p> <p style="text-align: center;"><math>12x^2 + 3x + 2</math></p> $x - 3 \overline{)4x^3 + 0x^2 + 3x + 2}$ <p style="text-align: center;"><math>4x^2 + 12x \leftarrow</math></p> $x - 3 \overline{)4x^3 + 0x^2 + 3x + 2}$ <p style="text-align: center;"><math>-4x^3 + 12x^2</math></p> <p style="text-align: center;"><math>12x^2 + 3x + 2</math></p> | <p>Original problem</p> <p>Set up division problem in long division format. The dividend needs add an <math>x^2</math> term and it needs to be put in standard form from highest to lowest degree.</p> <p>Divide first term of divisor, <math>x</math>, into first term of dividend, <math>4x^3</math>, put result in quotient area.</p> <p>Multiply the term you just put in the quotient area, <math>4x^2</math> times the divisor and put result below dividend.</p> <p>Subtract this new polynomial by first changing signs. See <b>red</b> color for signs.</p> <p>Secondly, finish the subtraction by combining terms in each column.</p> <p>Bring down the rest of the dividend and now you will be dividing with a new dividend of: <math>12x^2 + 3x + 2</math></p> <p>Divide first term of divisor, <math>x</math>, into first term of new dividend, <math>12x^2</math>. Put the result, <math>12x</math>, in the quotient area.</p> |
|---|---|

$$\begin{array}{r}
 4x^2 + 12x \\
 x - 3 \overline{) 4x^3 + 0x^2 + 3x + 2} \\
 \underline{- 4x^3 + 12x^2} \phantom{+ 2} \\
 12x^2 + 3x + 2 \\
 \underline{12x^2 - 36x} \phantom{+ 2} \leftarrow
 \end{array}$$

Multiply the term which was just put in the quotient area,  $12x$ , times the divisor and put the result below the dividend.

$$\begin{array}{r}
 4x^2 + 12x \\
 x - 3 \overline{) 4x^3 + 0x^2 + 3x + 2} \\
 \underline{- 4x^3 + 12x^2} \phantom{+ 2} \\
 12x^2 + 3x + 2 \\
 \underline{- 12x^2 + 36x} \phantom{+ 2} \\
 39x \phantom{+ 2} \leftarrow
 \end{array}$$

Subtract this new polynomial by changing signs and combining terms in each column.

$$\begin{array}{r}
 4x^2 + 12x \\
 x - 3 \overline{) 4x^3 + 0x^2 + 3x + 2} \\
 \underline{- 4x^3 + 12x^2} \phantom{+ 2} \\
 12x^2 + 3x + 2 \\
 \underline{- 12x^2 + 36x} \phantom{+ 2} \downarrow \\
 39x + 2
 \end{array}$$

Bring down the rest of the last divided and now you will have a new dividend.

$$\begin{array}{r}
 4x^2 + 12x + 39x \leftarrow \\
 x - 3 \overline{) 4x^3 + 0x^2 + 3x + 2} \\
 \underline{- 4x^3 + 12x^2} \phantom{+ 2} \\
 12x^2 + 3x + 2 \\
 \underline{- 12x^2 + 36x} \phantom{+ 2} \\
 +39x + 2
 \end{array}$$

Divide first term of divisor,  $x$ , into first term of new dividend,  $39x$ .

$$\begin{array}{r}
 4x^2 + 12x + 39x \\
 x - 3 \overline{) 4x^3 + 0x^2 + 3x + 2} \\
 \underline{- 4x^3 + 12x^2} \phantom{+ 2} \\
 12x^2 + 3x + 2 \\
 \underline{- 12x^2 + 36x} \phantom{+ 2} \\
 39x + 2 \\
 39x - 117 \leftarrow
 \end{array}$$

$$\begin{array}{r}
 4x^2 + 12x + 39x \\
 x - 3 \overline{) 4x^3 + 0x^2 + 3x + 2} \\
 \underline{- 4x^3 + 12x^2} \phantom{+ 2} \\
 12x^2 + 3x + 2 \\
 \underline{- 12x^2 + 36x} \phantom{+ 2} \\
 39x + 2 \\
 \underline{- 39x + 117} \phantom{+ 2} \\
 119 \leftarrow
 \end{array}$$

$$(2 + 4x^3 + 3x) \div (x - 3) = 4x^2 + 12x + 39x + \frac{119}{x - 3}$$

Multiply the term you just inserted in the quotient area,  $39x$ , time the divisor and put result below your new dividend.

Subtract this new polynomial by changing signs and combining terms in each column.

You should see that the new dividend is just 119 and there is no other terms to bring down in the old dividend.

You should also see that the new dividend has a degree less than the divisor. The divisor has a degree of **one** and the new dividend has a degree of **zero**.

The division process is now finished, so write the polynomial in the quotient area along with the remainder over the divisor.