Introductions to Functions

In order to understand functions we first look at a **relation**.

A **relation** is a set of ordered pairs. Example:

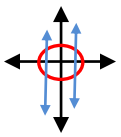
{ (2, 3), (4, 5), (8, 20) }

The set of *x*-coordinates is called the **domain** and the set of y-coordinates is called the **range**. In the relation: { (2, 3), (4, 5), (8, 20) }, the domain would be {2, 4, 8} and the range would be {3, 5, 20}.

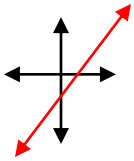
A **function** is a relation that assigns each *x*-value to exactly one y-value. This means that if you given a *x*-value like 10, it can only have one y-value. The equations we have done in this class for lines and parabolas are functions. If you recall whenever we substituted a *x*-value into an equation, we obtained just one y-value.

If you have a graph, you can tell if it is a function by the vertical line test.

Vertical Line Test: If a vertical line can be drawn so that it intersects a graph more than once, the graph is **not** a function.



Not a function because a vertical line intersected the graph more than once.



Is a function because a vertical line will not intersect the graph more than one once.

There is new notation to show a function. It usually uses the letter *f*, but it can also use *g* or *h*. We have been writing equations like $y = 3x + 4$ and in function notation it is written as:

$f(x) = 3x + 4$.

It is important to realize that $f(x)$ does not mean **f** times **x**. It means the "function of *x*" or "f of *x*". To evaluate a function you simply substitute in a value. Example:

Evaluate $f(2)$ for $f(x) = 3x + 4$.

$$f(2) = 3(2) + 4$$
$$= 6 + 4$$
$$= 10$$

Make sure you substitute the 2 into the $f(x)$ on the left side of the equation as well as the right side.

The function we just used, $f(x) = 3x + 4$, is called a function in symbolic representation. Functions can also be described in the following representations:

- Graphical
- Numerical
- Verbal
- Diagrammatical

The next page has problems in graphical and numerical form. Numerical form uses a table of values.

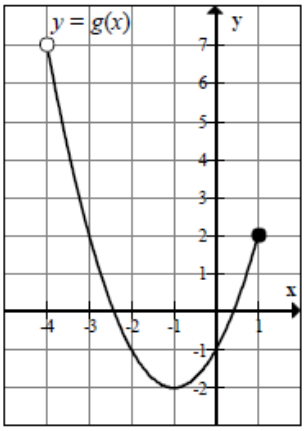


Figure 1.

x	$y = h(x)$
-7	-3
0	64
1	64
-5	-4
17	0
20	3

Figure 2.

Steps to Evaluate a Function for a Value

1. W.O.P.

2. If the function is given in symbolic form, that is, an equation is given, then substitute in the value for *x* and simplify. For example:

   Evaluate $f(6)$ for $f(x) = 3x + 4$.

   $$f(6) = 3(6) + 4$$
   $$= 18 + 4$$
   $$= 22$$

3. If a graph is given, the value in the ( ) is the *x* coordinate and find the corresponding *y* coordinate. For example, find *g*(-3) using Figure 1.. The *x* coordinate will be -3 and the *y* coordinate on the graph is 2. Thus *g*(-3) = 2.

4. If a table is given the value in the ( ) is the *x* value to be found in the *x* column and find the corresponding *y* value in the other column. For example, find *h*(20) using Figure 2. The *x* value is 20 and the value of 3 is found in the other column. Thus
   h(20) = 3.

Steps to Solve a Function Given an Output of the Function

1. W.O.P.

2. If the function is given in symbolic form, that is, an equation is given, then substitute the function definition for *f*(*x*) and solve for *x*. For example, solve *f*(*x*) = 10, given *f*(*x*) = 3*x* + 4.

Details for Solving	Notes
$f(x) = 10$ $(3x + 4) = 10$ $3x + 4 = 10$ $3x + 4 - 4 = 10 - 4$ $3x = 6$ $\frac{3x}{3} = \frac{6}{3}$ $x = 2$	Substitute in 3*x* + 4 for *f*(*x*) and solve.

The solution set is {2}.

3. If a graph is given, then the output value is the *y* coordinate; find the corresponding *x* coordinate. For example, solve $g(x) = 2$ using Figure 1. You will notice that the *y* coordinate of 2 appears twice at (-3, 2) and (1, 2) and there will be two *x* values. The solution is {-3, 1}.

4. If a table is given, then the output value is the *y* value. Find the corresponding *x* value. For example, solve $h(x) = -4$ using Figure 2. Find -4 in the right column and the corresponding *x* value is -7. The solution is {-7}.