Make sure all of the graphs, you make, have the following items:

- An ***x*** axis and a ***y*** axis with arrows and labeled with ***x*** and ***y***.
- Tick marks with numbers.

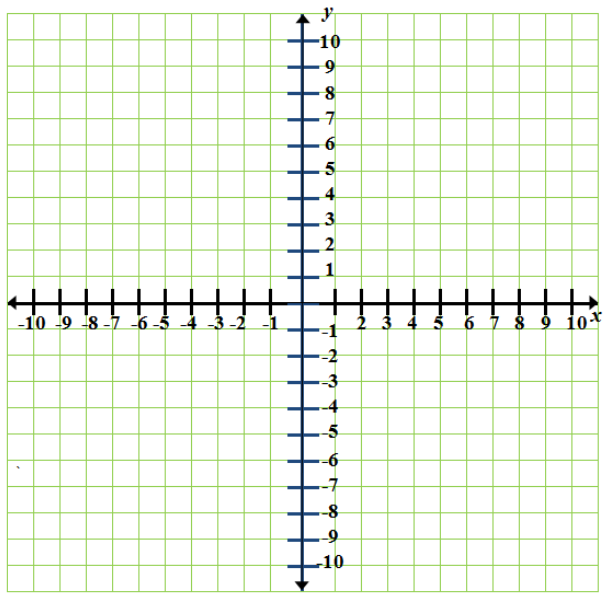


Figure 1: Example Grid with Axes, Tick Marks and Labels

If you are doing fractions like $\frac{1}{3}$, make sure you use three squares as one unit so that you do not have to put a point on a non-grid corner. In other words, **all points need to be on grid corners**. See Figure 2: Grid with Each Square Equal to $\frac{1}{3}$.

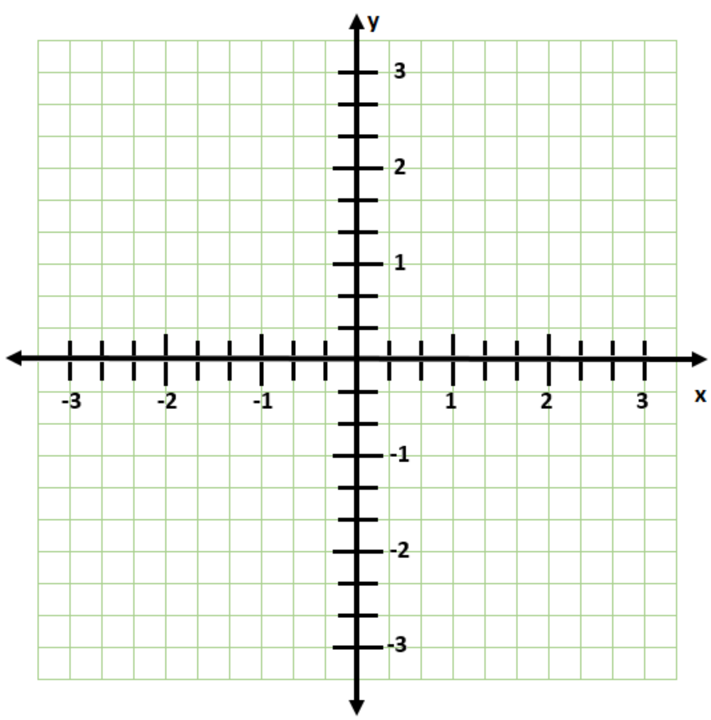
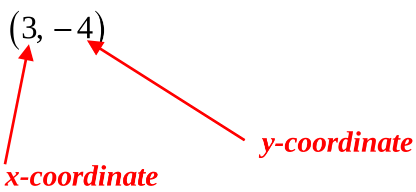


Figure 2: Grid with Each Square Equal to $\frac{1}{3}$

Points are made from an ordered pair. An **ordered pair** consists of two numbers in a pair of parenthesis. The numbers are separated by a comma. The first number is the ***x*-coordinate** and the second number is the ***y*-coordinate**.



To graph a point do the following:

1. Start at the origin, which is where the axes intersect, and then move left or right the number of units of the ***x***-coordinate. Right is + and left is - . For our example, the x-coordinate, 3, is positive so we move along the x-axis to 3.

2. After moving left or right, move up or down with the value of the ***y***-coordinate. Up is + and down is - . For our example we now move down 4 units since the *y*-coordinate, -4, is a negative number.

3. After moving up or down put a big dot on the point.

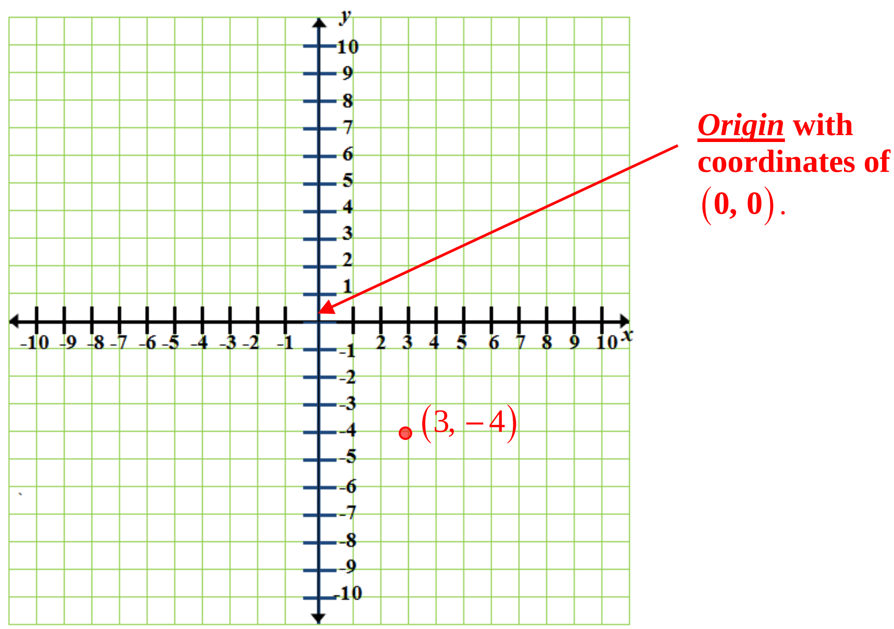


Figure 3: Graph of the Point, (3, - 4)

Be careful with plotting points by starting at the origin, then go left or right with the ***x***-coordinate and then up or down with the ***y***-coordinate. Remember ***x*** is before ***y*** alphabetically and remember the ***x***-coordinate is before the ***y***-coordinate in the ordered pair.

Equations:
We will use equations to generate points. Every equation with ***x*** or ***y*** or both ***x*** and ***y*** can be graphed on a coordinate system by plotting points and then connecting the points with a line.

We will now be dealing with two variables but many of the concepts remain the same. The first issue is checking a solution. If you recall, we checked solutions by substituting in values and then we checked to see if the equation was equal. We do the same thing when checking a solution for an equation with ***x*** and ***y***. The only difference is we substitute in values for **both** ***x*** and ***y***.

Note: The solution for an equation with ***x*** and ***y*** is the point where the graph intersects the point on your grid.

Steps For Determining If An Ordered Pair Is A Solution

1. Write out original problem.
2. Substitute in ***x*** and ***y*** using ( )'s and change = to, $\stackrel{?}{=}$.
3. Simplify each side of the equation.
4. Once each side has one number, change the $\stackrel{?}{=}$ to a = .
5. Write true or false by the last statement.
6. If "true, "write, "the ordered pair (… , ….) is a solution."
7. If "false, "write, "the ordered pair (… , ….) is not a solution."

Example:

Is (-2, 4) a solution for $2x + 3y = 12$?

$2(-2) + 3(4) \stackrel{?}{=} 12$
$-4 + 12 \stackrel{?}{=} 12$
$8 = 12$, false
The ordered pair (-2, 4) is not a solution.

Points are made by using ordered pairs. Any shape is made by plotting points and connecting the points. In this class we will concentrate on lines. A common method to graph a line is to use an **equation** to generate the points. An equation that makes a line is called a **linear equation**. The standard form of a linear equation is: $\underline{\boldsymbol{Ax + By = C}}$**,** where A, B and C are real numbers and A and B cannot both be 0. If they were, there would be no ***x*** or ***y*** terms.

We will learn three methods for graphing equations for lines.

1. **Table of Values**: This method involves picking values for ***x*** and then finding ***y***, writing ordered pairs in a table and then plotting the points and connecting them.
2. **Method of Intercepts**: This method involves finding the ***x*** and ***y*** intercepts and then connecting a line between the intercepts.
3. **Slope Intercept Method**. This method involves putting the equation in a particular form and then using the ***y***-intercept and slope to graph the line.

Steps for Graphing a Linear Equation with a Table of Values

1. Write out the original problem.
2. Solve the equation for the ***y*** variable so that the ***y*** variable is on the left and everything else is on the right side of the equation.
3. Write the new equation in five columns.
4. Pick two negative values, zero, and two positive values for ***x*** and substitute each one in a separate column.
5. Simplify the right side to get a value for ***y***.
6. Put the ***x*** and ***y*** values for each column into a table of ordered pairs.
7. Make a coordinate system.
8. Plot the five points with five big fat dots and label the coordinates.
9. Connect the dots with a line.

Example: Graph, $\boldsymbol{y = 2x - 3}$ using a table of values and five points. *Put results in separate table.*

$y = 2x - 3$ let $x = -2$ $y = 2(-2) - 3$ $y = -4 - 3$ $y = -7$	$y = 2x - 3$ let $x = -1$ $y = 2(-1) - 3$ $y = -2 - 3$ $y = -5$	$y = 2x - 3$ let $x = 0$ $y = 2(0) - 3$ $y = 0 - 3$ $y = -3$	$y = 2x - 3$ let $x = 1$ $y = 2(1) - 3$ $y = 2 - 3$ $y = -1$	$y = 2x - 3$ let $x = 2$ $y = 2(2) - 3$ $y = 4 - 3$ $y = 1$

(x, y)
(- 2, - 7)
(- 1, - 5)
(0, - 3)
(1, - 1)
(2, 1)

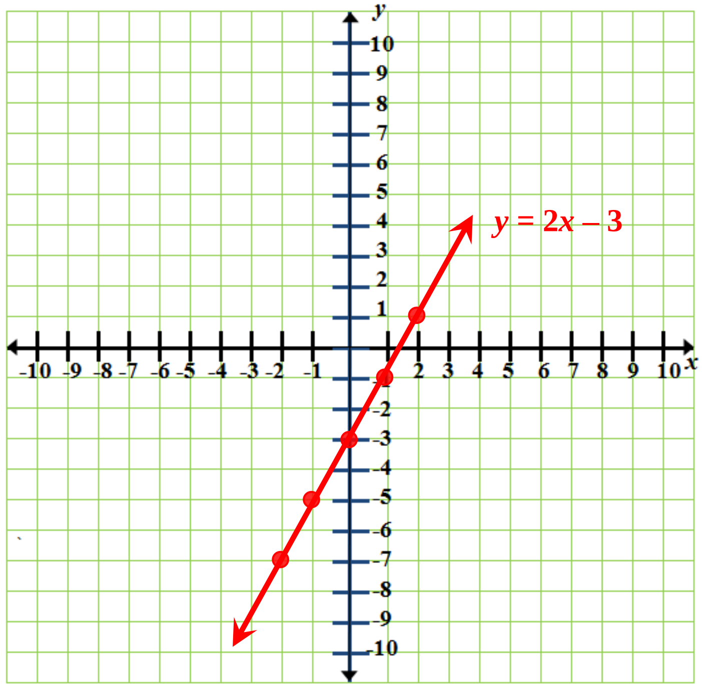


Figure 4: Graph of a Line Using a Table of Values

Steps for Graphing an Equation using a Intercepts

1. Write out original equation.
2. Set up three columns with the headings, ***x*-intercept**, ***y*-intercept** and **checkpoint**.
3. Write the original equation in each column.
4. In the 1st column substitute in $y = 0$ and solve the equation for x, in the 2nd column substitute in $x = 0$ and solve equation for y.
5. In the third column pick a value for x or y that is not zero and that is not the x or y intercept and solve for the remaining variable
6. Make a table with the three ordered pairs.
7. Make a coordinate system and plot these ordered pairs.
8. Connect the points with a line.

Example: Graph $3x – 5y = 15$.

***x*-intercept**	***y*-intercept**	**check point**
$3x – 5y = 15$ let $y = 0$ $3x – 5(0) = 15$ $3x – 0 = 15$ $3x = 15$ $\frac{3x}{3} = \frac{15}{3}$ $x = 5$	$3x – 5y = 15$ let $x = 0$ $3(0) – 5y = 15$ $0 – 5y = 15$ $-5y = 15$ $\frac{-5y}{-5} = \frac{15}{-5}$ $y = -3$	$3x – 5y = 15$ let $x = -5$ $3(-5) – 5y = 15$ $– 15 – 5y = 15$ $\mathbf{– 15 + 15 – 5y = 15 + 15}$ $-5y = 30$ $\frac{– 5y}{– 5} = \frac{30}{– 5}$ $y = – 6$

(x,y)
(5, 0)
(0, -3)
(-5, -6)

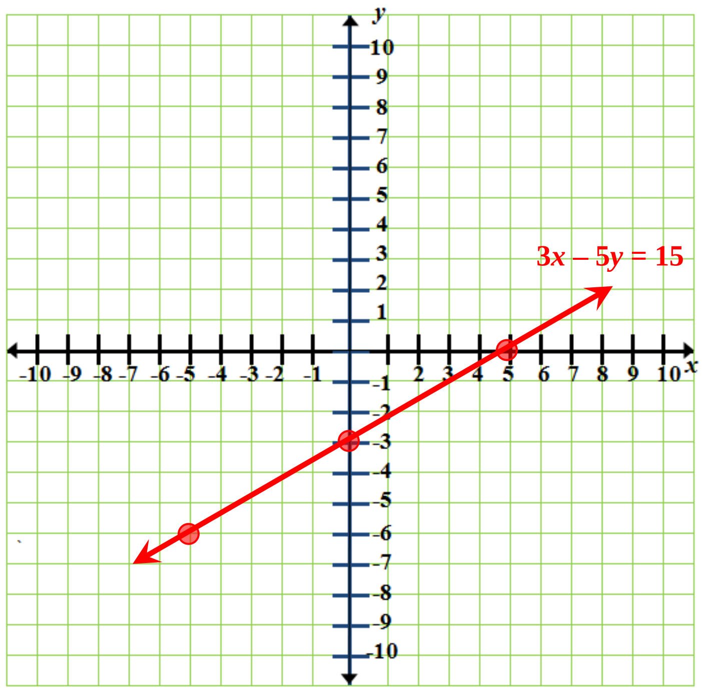


Figure 5: Graph of a Line Using the Method of Intercepts

Graphing Horizontal and Vertical Lines

Equations with just an ***x*** variable are graphed by solving for ***x*** and then graphing a vertical line.

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

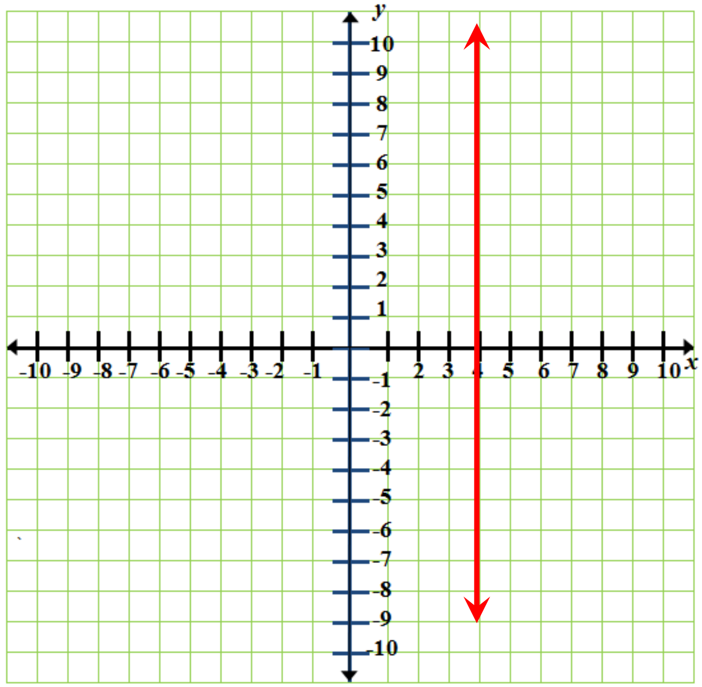


Figure 6: Graph of a Vertical Line

Equations with just a ***y*** variable are graphed by solving for ***y*** and then graphing a horizontal line.

$$2y = 6$$

$$\frac{2y}{2} = \frac{6}{2}$$

$$y = 3$$

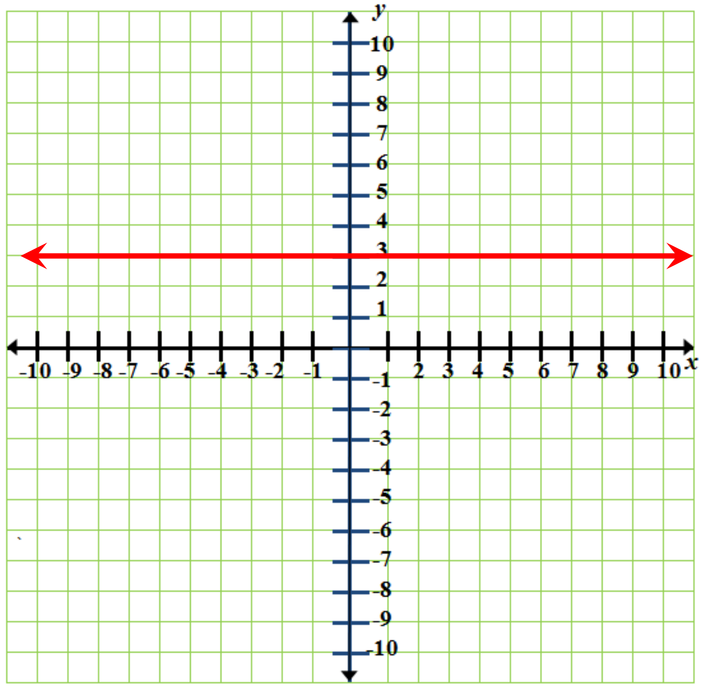


Figure 7: Graph of a Horizontal Line

No tables are necessary, just solve and graph.

Slope: We often use the word slope in everyday language. We say that something has a gradual slope or a steep slope. When we discuss slope we are talking about the steepness of a road, ramp or some similar object. For example, most of us have seen road signs that discuss the grade of a road, this is a measure of slope. In describing the road grade a % is used. In the Olympics, the announcers would talk about the ski slope using the degree of the slope.

This slope tells us how "steep" a line may be. Additionally, the slope in math tells the direction of the slope, that is, is the line going up from left to right or down from left to right. If the line is going up or rising from left to right we have a positive slope. If the line is going down or falling from left to right, we have a negative slope. If a line is horizontal, the slope is zero. If a line is vertical, the slope is undefined.

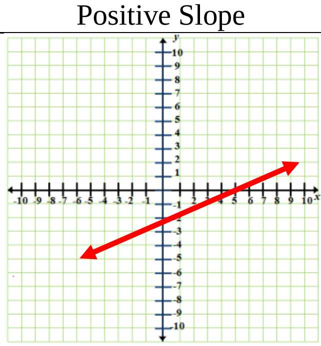
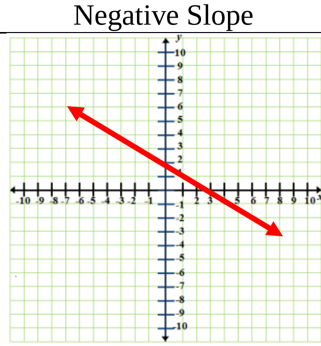
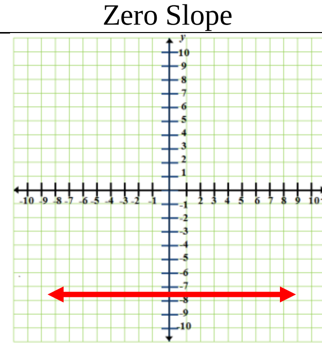
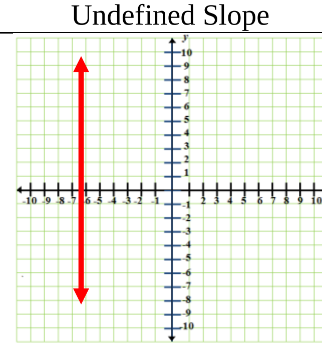
Positive Slope	Negative Slope	Zero Slope	Undefined Slope
			

Figure 8: Four Types of Slopes

In algebra we use a **fraction** to describe the slope. The numerator describes the change of y or the rise and the denominator describes the change in x or the run. A formula is used to describe or find the slope. This formula is called the slope formula.

Slope Formula: Given two points on a line, (x_1, y_1) and (x_2, y_2) the formula is,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

NOTE: A variable with a number to the right and lower is called a subscripted variable. The subscript is part of the identification for the variable. Thus an x_1 is different from an x_2.

Steps for Finding a Slope Given Ordered Pairs for Two Points

1. W.O.P.
2. Underneath the 1st pair write, $x_1 \quad y_1$, and underneath the 2nd pair write, $x_2 \quad y_2$.
3. Write out the slope formula, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
4. Substitute in the values for the variable, make sure you use ( )'s.
5. Simplify the fraction completely.

Example: Find the slope for points (3, -1) and (-4, 5).

Notes	Steps
Write out the points, and underneath 1st pair write, $x_1 \quad y_1$, and underneath the 2nd pair write, $x_2 \quad y_2$.	$(3,\ -1)$ and $(-4,\ 5)$ $x_1 \quad y_1 \qquad\quad x_2 \quad y_2$
Write out the slope formula.	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
Do the substitution.	$m = \dfrac{(5)-(-1)}{(-4)-(3)}$
	$m = \dfrac{5 + 1}{-4 - 3}$
Simplify the fraction.	$m = \dfrac{6}{-7}$
The sign of slope is negative which means the line "falls" from left to right. It falls at a rate of 6 units down for every 7 units to the right.	$m = -\dfrac{6}{7}$

If a slope has a value of zero in the denominator, the value of the slope is undefined. For example,

$$m = \frac{3}{0}$$

m is undefined

If a slope has a value of zero in the numerator and a non-zero value in the denominator, the value of the slope is zero. For example,

$$m = \frac{0}{8}$$

$$m = 0$$

Steps to graph a linear equation using the slope intercept method

1. Write out original problem.
2. Solve equation in terms of ***y*** so that it is in the form of:
   y = *m*x + *b*
3. After the equation is in this form add a **;** after the equation and write the slope-intercept form. Example
   $y = \frac{2}{3}x - 5\,;\;\; y = mx + b$
4. Write the following three lines:
   m = ___, the slope *(NOTE: if the value of m is an integer, put it over 1 to make it a fraction.)*
   b = _____, the ***y***-coordinate of the ***y*** intercept
   (0, _____), the ***y*** intercept.
5. Make a coordinate system and put a dot on the point for the ***y*** intercept.
6. Take out another color and then go up if slope is positive or down if slope is negative and make a ray that has the length of the value of the numerator of the slope. The ray starts at the ***y*** intercept and goes up or down the ***y***-axis. Write the number of units up or down by the ray.
7. Take out a third different color and make a horizontal ray from the end of the ray on the ***y***-axis. This ray always goes to the right and the length is the value of the denominator of the slope. Write the value by the ray.
8. At the end of the horizontal ray please put a point. Repeat steps six and seven to make another pair of vertical and horizontal rays and make a third point. You can continue this process to make more points.
9. Connect these three dots and write the original equation by the line.

Example:

$$\mathbf{2x + 3y = 6}$$

$$\mathbf{2x - 2x + 3y = 6 - 2x}$$

$$\mathbf{3y = -\,2x + 6}$$

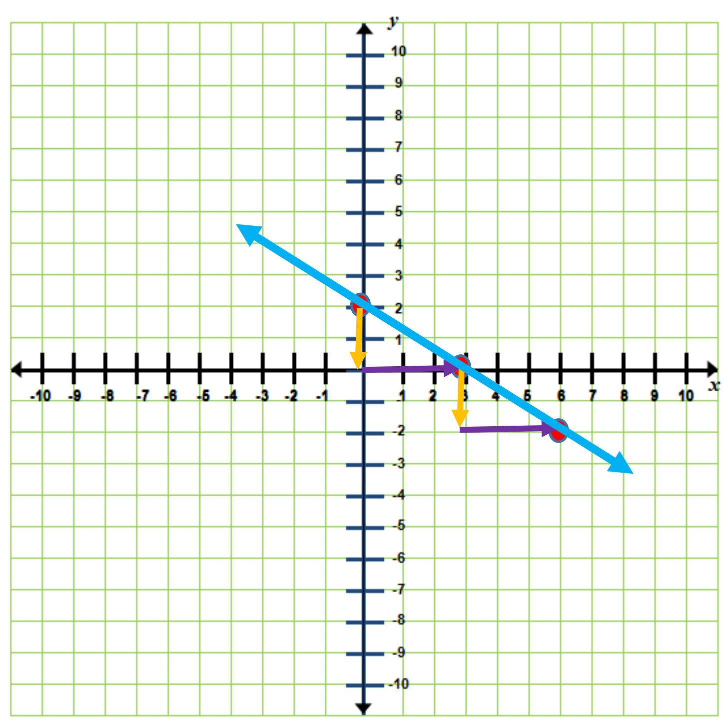
$$\mathbf{\frac{3y}{3} = \frac{-\,2}{3}x + \frac{6}{3}}$$

$$\mathbf{y = -\frac{2}{3}x + 2\,;\; y = mx + b}$$

$$\mathbf{m = -\frac{2}{3}}\text{, } \textbf{ the slope}$$

b = 2, the y – coordinate of the y intercept

(0, 2), the y intercept point



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