SUBTRACTION OF SIGNED NUMBERS

7 - 5 = 2 "Seven subtract five is two"
because 5 + 2 = 7

Subtraction on a number line is more difficult than addition.
To subtract 7 - 5 on a number line

THINK: 5 + ____________ = 7 Minuend - Subtrahend = Difference
(the answer) Subtrahend + Difference = Minuend

NOW using this addition problem, think how you add on the number line:

1. Start at 0 and move the distance and direction of the first addend.

2. From this point move the distance and direction of the second addend.

3. Where you stop is the sum. The sum is 7.

5 + _____ = 7

The answer is 7 - 5 is the distance (how far) and the direction (which way) we move to go from 5 to 7.

There are 2 places from 5 to 7.
We move in a positive direction.
So the answer must be 2.

Look carefully at these two problems:

\[
\begin{align*}
\text{1st no.} & - \text{2nd no.} = \text{1st no.} + \text{opposite of 2nd no.} \\
7 & - 5 = 7 + (-5) = 2
\end{align*}
\]

Instead of subtracting 5 from 7, we can get the same answer by adding the opposite of 5 to 7.
We will use this rule to subtract signed numbers.
-8 - 3 "negative eight subtract (positive) three"
is -8 + (-3) "negative eight add the opposite of three"

-11 The rule for adding with the same signs was used.

Let's look at subtracting this on the number line.
The answer, \( n \), of \(-8 - 3 = n\) is the number that can be added to 3 to get -8.

\[
3 + n = -8 \quad \text{(Isn't this the way to check subtraction?)}
\]

3 + n = -8

\[ \begin{array}{c}
0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \\
\hline
-12 \quad -11 \quad -10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \\
\end{array} \]

We start at 0 and move 3 places to the right.
We know that we must stop at -8, so \( n \) must tell the distance and direction that we must move to get to -8. The move is eleven places to the left, so the answer is -11. (It is much easier to use the subtraction rule.)

You must be able to read the problem so you'll know the operation and the sign of each number.

The operation symbol will be immediately after the first number. If there is not a negative sign AFTER the operation symbol, the next number is positive.

a. \(-5 - 4\) "negative five subtract four"
   (four is positive)
b. \(-6 - (-8)\) "negative six subtract negative eight"
   (the second number is negative)
c. \(4 - (-3)\) ",(positive) four subtract negative three"
d. \(3 - 12\) "(positive) three subtract (positive) twelve"

Instead of saying "subtract", you may say "minus" for the operation, but do not read -4 as "minus 4," say, "Negative 4." The way you read these problems can help you or hinder you, so learn to read them correctly. Using the subtraction rule, we will find the answers.

**SUBTRACTION RULE:**

1. Write the problem
2. On the next line use the subtraction rule:
   a. write the same first number
   b. change the subtraction symbol to addition
   c. change the second number to its opposite
3. Now you have an addition problem, ask yourself, "Do the addends have signs that are the same or that are different?"
   a. If the signs are the same:
      1. add absolute values
      2. use the sign of the addends in the sum
   b. If the signs are different:
      1. subtract the smaller absolute value from the larger absolute value
      2. use the sign of the number with the larger absolute value
   (REMEMBER absolute value just means distance)

   You may wish to review absolute value and the addition rules in your textbook.

   EXAMPLES:
   a. -5 - 4  b. -6 - (-8)  c. 4 - (-3)  d. 3 - 12
      -5 + (-4)  -6 + 8  4 + 3  3 + (-12)
      -9        2       7       -9

   There may be more than two numbers in a problem. If that is the case, remember the operation symbol will be immediately after each number.

   8 - 12 - (-4) - (-2) - 7

   This problem says:

   8 subtract (positive) 12 subtract negative 4 subtract negative 2 subtract (positive) 7

   8 - 12 - (-4) - (-2) - 7

   The operations are circled on the line above.
   To use the subtraction rule, you will change each of these subtractions (-) to addition (+). At the same time you will change the sign of the number after that subtraction to get its opposite.

   8  - 12  - (-4)  - (-2)  - 7
       \underline{+} 12 \underline{+} \underline{-} (-4) \underline{-} \underline{(-2)} \underline{-} 7
   8  + (-12)  + 4  + 2  + (\underline{(-7)})

   Now you have an addition problem.

   You may add from left to right; or since you now have only addition, you may apply the Commutative and Associative Properties of Addition and add numbers with the same signs then add the numbers with different signs. Finally, add those two sums for the answer.

   REMEMBER: you cannot change the order or grouping of subtraction, so do not use these properties until you have changed every subtraction to addition.
USING PROPERTIES OF ADDITION:

\[ 8 - 12 - (-4) - (-2) - 7 \]
\[ 8 + (-12) + 4 + 2 + (-7) \]
\[ (8 + 4 + 2) + [(-12) + (-7)] \]
\[ 14 + (-19) \]
\[ -5 \]

In this method, the subtraction rule is used to get addition first! This is not the first time you have used a different operation to find an answer.

To divide \( \frac{5}{8} \div \frac{3}{7} \), we used this rule:

1. keep the same first number
2. change division to multiplication
3. change the second number to its reciprocal
4. you now have a multiplication problem so use the rule for multiplication!

\[
\frac{5}{8} \div \frac{3}{7} = \frac{5}{8} \cdot \frac{7}{3} = \frac{35}{24} \text{ or } \frac{11}{24}
\]

In Subtraction

\[-8 - (-4)\]

is equivalent to

\[-8 + 4\]

To subtract we write an equivalent addition problem.

In Division

\[\frac{4}{9} \div \frac{2}{5}\]

is equivalent to

\[\frac{4}{9} \cdot \frac{5}{2}\]

To divide we write an equivalent multiplication problem.

**PROBLEMS:** Take three lines to do these problems.

1. Write the original problem on the first line.
2. Write the equivalent subtraction problem on the second line.
3. Write the answer on the third line.

I. Write the subtraction rule for signed numbers.
II. Subtract:

1. $-6 - 9$  
2. $8 - 15$  
3. $-7 - (-2)$  
4. $9 - (-6)$  

5. $-8 - (-12)$  
6. $3 - (-8)$  
7. $7 - 4$  
8. $-5 - 5$  

9. $-8 - (-8)$  
10. $-3 - 4 - (-6)$  
11. $3 - (-7) - 8$  

**ANSWERS:**

1. $1$st number $- 2$nd number  
   Same $1$st number $+$ opposite of $2$nd number  
   (Then use the appropriate addition rule.)

II.  
1. $-15$  
2. $-7$  
3. $-5$  
4. $15$  
5. $4$  
6. $11$  
7. $3$  
8. $-10$ (this becomes $-5 + (-5)$)  
9. $0$ (this becomes $-8 + 8$)  
10. $-1$  
11. $2$