Section 8.1

Confidence Interval for a single proportion
The point estimate for \( p \), the population proportion of successes, is given by the proportion of successes in a sample

\[
\hat{p} = \frac{x}{n}
\]

(Read as p-hat)

\( \hat{q} \) is the point estimate for the proportion of failures where

\[
\hat{q} = 1 - \hat{p}
\]

If \( np \geq 5 \) and \( nq \geq 5 \) the sampling distribution for \( \hat{p} \) is normal.
The maximum error of estimate, $E$, for a $x$-confidence interval is:

$$E = z_c \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

A $c$-confidence interval for the population proportion, $\hat{p}$, is

$$\hat{p} - E < p < \hat{p} + E$$
In a study of 1907 fatal traffic accidents, 449 were alcohol related. Construct a 99% confidence interval for the proportion of fatal traffic accidents that are alcohol related.

1. The point estimate for $p$ is
   \[ \hat{p} = \frac{x}{n} = \frac{449}{1907} = 0.235 \]
   \[ \hat{q} = 1 - 0.235 = 0.765 \]

2. $1907(.235) \div 5$ and $1907(.765) \div 5$, so the sampling distribution is normal.

3. \[ E = z_c \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.575 \sqrt{\frac{(0.235)(0.765)}{1907}} = 0.025 \]
In a study of 1907 fatal traffic accidents, 449 were alcohol related. Construct a 99% confidence interval for the proportion of fatal traffic accidents that are alcohol related.

Left endpoint

\[ \hat{p} - E = 0.235 - 0.025 = 0.21 \]

Right endpoint

\[ \hat{p} + E = 0.235 + 0.025 = 0.26 \]

\[ 0.21 < p < 0.26 \]

With 99% confidence, you can say the proportion of fatal accidents that are alcohol related is between 21% and 26%.
If you have a preliminary estimate for $p$ and $q$, the minimum sample size given a $x$-confidence interval and a maximum error of estimate needed to estimate $p$ is:

$$n = \hat{p}\hat{q}\left(\frac{Z_c}{E}\right)^2$$

If you do not have a preliminary estimate, use 0.5 for both $\hat{p}$ and $\hat{q}$. 
You wish to estimate the proportion of fatal accidents that are alcohol related at a 99% level of confidence. Find the minimum sample size needed to be accurate to within 2% of the population proportion.

With no preliminary estimate use 0.5 for $\hat{p}$ and $\hat{q}$

$$n = \hat{p}\hat{q}\left(\frac{Z_c}{E}\right)^2 = (0.5)(0.5)\left(\frac{2.575}{0.02}\right)^2 = 4414.14$$

You will need at least 4415 for your sample.
You wish to estimate the proportion of fatal accidents that are alcohol related at a 99% level of confidence. Find the minimum sample size needed to be accurate to within 2% of the population proportion. Use a preliminary estimate of $p = 0.235$.

\[ n = \hat{p}q \left( \frac{Z_c}{E} \right)^2 \]

\[ n = (0.235)(0.765)\left( \frac{2.575}{0.02} \right)^2 = 2980.05 \]

With a preliminary sample you need at least $n = 2981$ for your sample.
Hypothesis testing for
A single proportion
Test for Proportions

\( p \) is the population proportion of successes. The test statistic is \( \hat{p} = \frac{x}{n} \).

(the proportion of sample successes)

If \( np \geq 5 \) and \( nq \geq 5 \) the sampling distribution for \( \hat{p} \) is normal.

The standardized test statistic is:

\[
z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}
\]
A communications industry spokesperson claims that over 40% of Americans either own a cellular phone or have a family member who does. In a random survey of 1036 Americans, 456 said they or a family member owned a cellular phone. Test the spokesperson’s claim at $\alpha = 0.05$. What can you conclude?

1. Write the null and alternative hypothesis.

$$H_0: p \leq 0.40 \quad H_a: p > 0.40 \text{ (claim)}$$

2. State the level of significance.

$$\alpha = 0.05$$
3. Determine the sampling distribution.

1036(.40) > 5 and 1036(.60) > 5. **The sampling distribution is normal.**

4. Find the critical value.

5. Find the rejection region.

6. Find the test statistic and standardize it.

\[
n = 1036 \quad x = 456 \\
\hat{p} = \frac{x}{n} = \frac{456}{1036} = .44
\]

\[
z = \frac{.44 - .40}{\sqrt{(.40)(.60)/1036}} = \frac{0.04}{0.01522} = 2.63
\]

7. Make your decision.

\( z = 2.63 \) falls in the rejection region, so reject \( H_0 \)

8. Interpret your decision.

There is enough evidence to support the claim that over 40% of Americans own a cell phone or have a family member who does.