Portfolio

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Purpose

Throughout this course, you will work through a few prompts and collect them together in a course portfolio. These prompts represent key topics in this course. As you progress through the course, you will add to this portfolio and also have an opportunity to go back and edit any previous responses. By the end of the term, you will have a portfolio of work that demonstrates your understanding of this course.

Drafts

Each week, you will submit a draft of a portfolio prompt. Each draft may be submitted as many times as you like. Each draft will receive written feedback to help you edit your next submission. When a submission is polished, the feedback of "great!" will be written at the top of the page. Unless an issue is a minor fix, you will need to rewrite the submission, addressing all of the feedback you received. When submitting a draft,

- Write "Prompt #" at the top of the page, along with your name.
- Write out the entire instructions of the prompt before responding.
- Show all work, and write a sentence to justify anything that is unclear.
- Work vertically – do not write $a = b = c = d$ in a horizontal manner.
- At the end, respond to the prompt with a complete sentence.
- Any new prompt will begin on a new page.
- Any portfolio prompt with a "great!" on it should be saved to submit at the end of the term.

Full Portfolio

This portfolio will be worth 100 points at the end of the term. When you submit your portfolio, create a Cover Sheet. It should say "Portfolio" along with your name. Then include all of the portfolio prompts, in order, and staple your work in the top-left corner.

Prompts

Prompt 1

Consider the linear system below.

$$\begin{cases} 2x - 2y - 4z = -12 \\ x \qquad\quad - \ z = 1 \\ x + \ y + 2z = 10 \end{cases}$$

Solve the system using Gauss-Jordan elimination. Show all row reduction steps, making it clear which elementary row operation you are using.

Prompt 2

Consider the linear system below.

$$\begin{cases} x - 7y \qquad + 6\omega = 5 \\ \qquad\qquad z - 2\omega = -3 \\ -x + 7y - 4z + 2\omega = 7 \end{cases}$$

1. Write the system of equations as a vector equation.
2. Write the system as a matrix equation $A\mathbf{x} = \mathbf{b}$.
3. Solve the system of equations using linear algebra techniques. Indicate your solution in parametric vector form.

Prompt 3

Define the following $\mathbb{R}^3$ vectors.

$$\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \quad , \quad \mathbf{v} = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} \quad , \quad \mathbf{w} = \begin{bmatrix} -2 \\ 0 \\ -8 \end{bmatrix} \quad , \quad \mathbf{b} = \begin{bmatrix} -1 \\ -11 \\ -6 \end{bmatrix}$$

1. Is $\mathbf{b} \in \text{span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$?
2. Is $\{\mathbf{u}.\mathbf{v}, \mathbf{w}\}$ linearly independent or linearly dependent? If the set is linearly dependent, write a linear dependence relation.

Prompt 4

Find the standard matrix A for the linear transform $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that T first reflects points through the horizontal x_1-axis, secondly rotates points by an angle of $\frac{-\pi}{3}$ about the origin, and lastly reflects points through the line $x_2 = x_1$.

Prompt 5

Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. Find A^{-1}. Do not use Cramer's Method or the adjugate/adjoint method to find A^{-1}.

Prompt 6

Let $A = \begin{bmatrix} 1 & 3 & -4 & 2 & -1 & 6 \\ 3 & 9 & -12 & 7 & 1 & 15 \\ -3 & -9 & 13 & -10 & 6 & -15 \\ 1 & 3 & -3 & -1 & 6 & 6 \end{bmatrix}$.

1. What does it mean for a vector $\mathbf{x}$ to be in $\operatorname{Col} A$?
2. What does it mean for a vector $\mathbf{x}$ to be in $\operatorname{Nul} A$?
3. Find a basis for $\operatorname{Col} A$.
4. Find a basis for $\operatorname{Nul} A$.
5. Find a basis for $\operatorname{Row} A$.
6. Find a basis for $\operatorname{LNul} A$.
7. Find $\dim \operatorname{Col} A$, $\dim \operatorname{Nul} A$, $\dim \operatorname{Col} A^T$, $\dim \operatorname{Nul} A^T$.
8. Find $\operatorname{rank} A$.

Prompt 7

Let $A \in M_{3\times 3}$ and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ as defined below.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \quad , \quad \mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad , \quad \mathbf{v} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \quad , \text{ and } \quad \mathbf{w} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}.$$

1. Show that the characteristic polynomial for A is $-\lambda^3 + \lambda^2 + 4\lambda - 4$.
2. Find the eigenvalues of A (Hint: Use factor-by-grouping).
3. Find a basis for the eigenspace of each eigenvalue.
4. Diagonalize A – that is, find two matrices, D and P, such that $A = PDP^{-1}$. The matrix D must be a diagonal matrix.
5. Find $A\mathbf{u}, A\mathbf{v}$, and $A\mathbf{w}$ without using a matrix-vector product.

Prompt 8

Let $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ as defined below.

$$\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad , \quad \mathbf{v} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \quad , \text{ and } \quad \mathbf{w} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}.$$

1. Find $\mathbf{u} \cdot \mathbf{w}$.
2. Determine if $\mathbf{u}$ and $\mathbf{v}$ are orthogonal or not. Show any work to support your conclusion.
3. Normalize $\mathbf{u}$.