MTH 253Z
Midterm Review

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1. Evaluate $\lim_{n \to 0} (1 - \cos n)^{n^2}$.

2. Find the linearization to $f(x) = \sec^2 x$ at $a = \frac{\pi}{6}$.

3. Find the linear approximation to $g(t) = \ln(t^2 + 1)$ at $a = 1$.

4. Use a linear approximation to estimate $\sqrt[4]{16.05}$. Express your conclusion as a fraction.

5. Provided are the graphs of $f(x) = \sqrt[3]{x - 1}$ (in red) and $g(x) = e^{-x} + 1$ (in blue). Use x_3 in Newton's method to approximate the solution to $f(x) = g(x)$. Show all computations by hand, and use a calculator to evaluate those computation. Round your conclusion to the nearest thousandth.

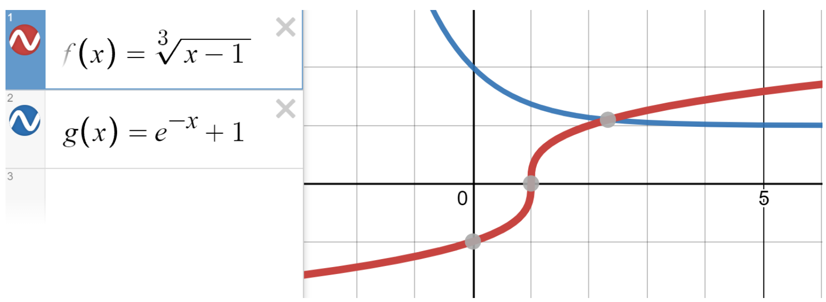

Figure 1: Graphs of $y = f(x)$ and $y = g(x)$.

6. Let $f(x) = \arctan(x^2 - 2)$. Beginning with $x_1 = 1$, use x_4 in Newton's method to approximate the roots of f. Do all computations by hand, and show all of your work to support your conclusion. Round your conclusion to the nearest thousandth.

7. Find a formula for the nth term of the sequence, a_n.

$$\left\{ \frac{2}{1}, \frac{4}{4}, \frac{8}{9}, \frac{16}{16}, \frac{32}{25}, \frac{64}{36}, \ldots \right\}$$

8. Determine if the sequence converges or diverges. If it converges, find its limit. Show all work to justify your conclusion.

$$\left\{ \frac{(-1)^n}{2\sqrt{n}} \right\}_{n=5}^{\infty}$$

9. Determine if the sequence converges or diverges. If it converges, find its limit. Show all work to justify your conclusion.

$$\left\{15, -18, \frac{108}{5}, \frac{-648}{25}, \frac{3888}{125}, \ldots\right\}$$

10. Sketch the graph of the sequence provided below on a Cartesian plane. Plot the first ten terms of the sequence.

$$\{2(0.9)^{n-1}\}_{n=1}^{\infty}$$

11. Suppose a sequence $\{a_n\}_{n=1}^{\infty}$ is defined by $a_n = \dfrac{2^n - 1}{n!}$. Respond to the following questions.

    (a) Is $\{a_n\}$ monotonic? If so, is it increasing or decreasing?

    (b) Is $\{a_n\}$ bounded? If so, provide both an upper and a lower bound.

    (c) Is $\{a_n\}$ convergent or divergent? If it is convergent, what is its limit?

12. Determine whether the series is convergent or divergent by expressing the nth partial sum as a telescoping series. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$$

13. Determine whether the series is convergent or divergent. If it is convergent, find its sum. Justify your conclusion as specifically as possible.

$$4 + 3 + \frac{9}{4} + \frac{27}{16} + \cdots$$

14. Determine whether the series is convergent or divergent. Justify your conclusion as specifically as possible.

$$\sum_{n=1}^{\infty} \frac{3}{\sqrt[6]{n^5}}$$

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$$\sum_{n=1}^{\infty} \frac{3}{\sqrt[5]{n^6}}$$

16. Consider the series below.

$$\sum_{n=1}^{\infty} \frac{2}{1+n^2}$$

    (a) Show that the series converges. Show all work, and support your conclusion as clearly as possible.

    (b) Approximate the series with s_5. Round your conclusion to the nearest hundredth.

    (c) How many terms are necessary to approximate s to the nearest thousandth?