MTH 253Z
Final Review

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1. Determine whether the following series converges or diverges. If it converges, find its sum.

$$\frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \cdots$$

2. Determine whether the following series converges or diverges. If it converges, find its sum. Justify your conclusion as specifically as possible.

$$686 + 588 + 504 + 432 + \cdots$$

3. Determine whether the following series converges or diverges. If it converges, find its sum. Justify your conclusion as specifically as possible.

$$432 + 504 + 588 + 686 + \cdots$$

4. Determine whether the following series converges or diverges. If it converges, find its sum. Justify your conclusion as specifically as possible. *Hint: Consider a partial fraction decomposition.*

$$\sum_{n=0}^{\infty} \frac{2}{(n+1)(n+2)}$$

5. Determine whether the following series converges or diverges. Justify your conclusion as specifically as possible.

$$\sum_{n=4}^{\infty} \frac{7}{n^{\frac{6}{7}}}$$

6. Determine whether the following series converges or diverges. Justify your conclusion as specifically as possible.

$$\sum_{n=4}^{\infty} \frac{e}{n^{\pi}}$$

7. Determine whether the following series converges or diverges. Justify your conclusion as specifically as possible.

$$\sum_{n=1}^{\infty} \frac{(n+1)(2n+1)(3n+1)}{n^3+n^2+n+1}$$

8. Determine whether the following series converges or diverges. Justify your conclusion as specifically as possible.

$$\sum_{n=2}^{\infty} \frac{(n+1)(2n+1)(3n+1)}{n^4-n^3-n^2-n-1}$$

9. Determine whether the following series converges or diverges. Justify your conclusion as specifically as possible.

$$\sum_{n=2}^{\infty} \frac{(n+1)(2n+1)(3n+1)}{n^4+n^3+n^2+n+1}$$

10. Determine whether the following series converges or diverges. Justify your conclusion as specifically as possible.

$$\sum_{n=1}^{\infty} \frac{(n+1)(2n+1)(3n+1)}{n^5+n^4+n^3+n^2+n+1}$$

11. Determine whether the following series converges or diverges. Justify your conclusion as specifically as possible.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^3}{n^4+1}$$

12. Determine whether the following series converges or diverges. Justify your conclusion as specifically as possible.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}5^{3n}}{\sqrt[3]{n}}$$

13. Express the function $f(x) = \dfrac{x^2}{2+x^3}$ as a the sum of a power series and find its radius of convergence.

14. Find the Taylor series for $f(x) = \left(\frac{2}{3}\right)^x$ centered at 1 and find its interval of convergence.

15. Use the binomial series to find the series expansion of $\dfrac{-2}{\sqrt[4]{32+2x}}$.

16. Use the binomial series to find the coefficient of the third-degree term in the series expansion of $\dfrac{-2}{\sqrt[4]{32+2x}}$.

17. Estimate the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^3}{n^4+1}$ with the fourth partial sum of the series.

18. How many terms must be used to approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^3}{n^4+1}$ to within 0.0001 of the true value of the sum? Use Desmos to support your conclusion.