

# Portfolio

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## Purpose

Throughout this course, you will work through a few prompts and collect them together in a course portfolio. These prompts represent key topics in this course. As you progress through the course, you will add to this portfolio and also have an opportunity to go back and edit any previous responses. By the end of the term, you will have a portfolio of work that demonstrates your understanding of this course.

## Drafts

Each week, you will submit a draft of a portfolio prompt. Each draft may be submitted as many times as you like. Each draft will receive written feedback to help you edit your next submission. When a submission is polished, the feedback of “great!” will be written at the top of the page. Unless an issue is a minor fix, you will need to rewrite the submission, addressing all of the feedback you received. When submitting a draft,

- Write “Prompt #” at the top of the page, along with your name.
- Write out the entire instructions of the prompt before responding.
- Show all work, and write a sentence to justify anything that is unclear.
- Work vertically – do not write  $a = b = c = d$  in a horizontal manner.
- At the end, respond to the prompt with a complete sentence.
- Any new prompt will begin on a new page.
- Any portfolio prompt with a “great!” on it should be saved to submit at the end of the term.

## Full Portfolio

This portfolio will be worth 100 points at the end of the term. When you submit your portfolio, create a Cover Sheet. It should say “Portfolio” along with your name. Then include all of the portfolio prompts, in order, and staple your work in the top-left corner.

## Prompts

### Prompt 1

Draw a number line, and place 0, 1, 2, 3, 4 and 5 on it. By the end of this prompt,  $\tan 4$  and  $\ln 3$  will be included on this number line.

1. Consider the function  $\tan x$ .
  - (a) Find the linearization of  $\tan x$  at  $\pi$ .
  - (b) Use the linearization to approximate the value of  $\tan 4$ .
  - (c) Place  $\tan 4$  on your number line.
  - (d) Is your approximation an over- or under-approximation for  $\tan 4$ ? Explain why in one or two sentences.
2. Consider the number  $\ln 3$ .
  - (a) Find a function and a value to produce a linear approximation that will be used to approximate  $\ln 3$ .
  - (b) Use your function to find a linear approximation for  $\ln 3$ .
  - (c) Place  $\ln 3$  on your number line.
  - (d) Is your approximation an over- or under-approximation for  $\ln 3$ ?

### Prompt 2

The equation  $x^5 + x^3 + 6 = 3x^4 + x^2 + x$  has 3 real roots. Use Newton's method to approximate these roots with  $x_3$ . Make initial approximations of  $x_1 = -1, 1, 3$ . Show all work to support your conclusions, and round your conclusions to the nearest hundredth. Please

### Prompt 3

Consider the sequence  $\{a_n\} = \{-250, 100, -40, 16, \frac{-32}{5}, \dots\}$ .

1. Determine whether the sequence  $\{a_n\}$  converges or diverges. If it converges, find  $\lim_{n \rightarrow \infty} a_n$ . Justify your conclusion as specifically as possible.
2. True or False: This sequence is monotonic.
3. True or False: This sequence is bounded.

### Prompt 4

Consider the series  $\sum a_n = -250 + 100 - 40 + 16 - \frac{32}{5} + \dots$ . Determine whether the series  $\sum a_n$  converges or diverges. If the series converges, find the sum of the series. Justify your conclusion as specifically as possible. Be sure to state what conditions are met that allow you to come to the conclusion that you come to.

### Prompt 5

Determine whether the series  $\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^3}$  converges or diverges. Justify your conclusion as specifically as possible. Be sure to state what conditions are met that allow you to come to the conclusion that you come to. *There is no need to find the sum if it converges.*

### Prompt 6

Determine whether the series  $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n-1}}{n^2 + 5n + 3}$  converges or diverges. Justify your conclusion as specifically as possible. Be sure to state what conditions are met that allow you to come to the conclusion that you come to. *There is no need to find the sum if it converges.*

### Prompt 7

Consider the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[5]{n^8}}$ .

1. Show that the series converges.
2. Approximate the sum of the series with  $s_4$ .
3. Determine the number of terms needed to estimate the sum with an error of at most 0.001.

### Prompt 8

Find the radius of convergence and the interval of convergence for the power series below.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n(x-4)^n}{n^3 + 1}$$