MTH 252Z Lab
Volume by Shells

Damien Adams

Prompts

1. Consider a solid of revolution with volume V. When should a shell method be used to find V? When should a washer method be used for finding V? Draw a solid to represent each of these three situations.

2. Let $\mathcal{R}$ be the region in the first quadrant enclosed by the curves $y = \sin x$, $y = \cos x$, and the y-axis. For each of the following prompts, you should include a sketch of the region/solid being considered, as well as a labeled typical disk or washer.

   (a) Let $\mathcal{S}_1$ be the solid obtained by rotating $\mathcal{R}$ about the y-axis. Write a definite integral that represents the volume of $\mathcal{S}_1$.

   (b) Let $\mathcal{S}_2$ be the solid obtained by rotating $\mathcal{R}$ about the line $x = -1$. Write a definite integral that represents the volume of $\mathcal{S}_2$.

3. Let $f(x) = 9 - x^2$. Let $\mathcal{S}$ be the solid obtained by rotating the region enclosed by the x-axis and $y = f(x)$ about the axis $x = -3$.

   (a) Which method(s) may be used to compute the volume of $\mathcal{S}$: Disk Method, Washer Method, and/or Shell Method?

   (b) Set up an integral that represents the volume of $\mathcal{S}$.

   (c) Find the volume of $\mathcal{S}$.

4. Consider the region $\mathcal{R}$ bounded by $y = \sin(x^2)$, $y = 0$, $x = 0$, and $x = \sqrt{\pi}$.

   (a) Find the area of $\mathcal{R}$.

   (b) Suppose we rotate $\mathcal{R}$ about the y-axis. What kind of solid is generated? *Come up with a real-life example rather than a mathematical shape.*

   (c) Suppose each unit represents one yard. How many cubic yards of clay would be needed to make this solid?

5. Consider the region enclosed by a semicircle of radius r (having equation $f(x) = \sqrt{r^2 - x^2}$) and the x-axis. Sketch this region. Sketch this solid and find the volume of this solid by the method of cylindrical shells.