MTH 252Z Lab
Volume

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Prompts

1. Consider a solid of revolution with volume V. When should a disk method be used to find V? When should a washer method be used for finding V? Draw a solid to represent each of these three situations.

2. Let $\mathcal{R}$ be the region in the first quadrant enclosed by the curves $y = \sin x$, $y = \cos x$, and the y-axis. For each of the following prompts, you should include a sketch of the region/solid being considered, as well as a labeled typical disk or washer.

   (a) Let $\mathcal{S}_1$ be the solid obtained by rotating $\mathcal{R}$ about the x-axis. Write a definite integral that represents the volume of $\mathcal{S}_1$.

   (b) Let $\mathcal{S}_2$ be the solid obtained by rotating $\mathcal{R}$ about the line $y = 2$. Write a definite integral that represents the volume of $\mathcal{S}_2$.

3. Let $f(x) = x^2 + 2$ and $g(x) = 4 - x^2$, and let $\mathcal{R}$ represent the region enclosed between $y = f(x)$ and $y = g(x)$. Let $\mathcal{S}$ be the "ring" obtained by rotating $\mathcal{R}$ about the x-axis.

   (a) Set up an integral that represents the area of $\mathcal{R}$.

   (b) Find the area of $\mathcal{R}$.

   (c) Set up an integral that represents the volume of $\mathcal{S}$.

   (d) Find the value of $\mathcal{S}$.

4. Consider the region enclosed by a semicircle of radius r (having equation $f(x) = \sqrt{r^2 - x^2}$) and the x-axis. Sketch this region. What is the solid of revolution obtained by rotating this region about the x-axis? Sketch this solid and find the volume of this solid.

5. Let T be the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$. Let V be the volume of the solid obtained by rotating T about the line $x = a$ with $a > 1$. Find V when

   (a) $a = 2$

   (b) $a = 3$

   (c) $a = 10$