MTH 252Z Lab
Area Between Curves

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Prompts

1. Let $f(x) = 2^x$ and $g(x) = -x^2 + 2x + 1$.
   (a) Use Desmos to graph both $f(x)$ and $g(x)$.
   (b) Using the graph, identify the two points of intersection for these two curves. Determine which curve is greater than the other between the intersection points.
   (c) Set up an integral that represents the area of the region enclosed by the curves $y = f(x)$ and $y = g(x)$.
   (d) Find the exact value of the area of the region between the two curves.
2. Find the exact value of the area of the region between the curves $y = \sqrt{x}$ and $y = x^3$.
3. On Desmos, graph both $x + y^2 = 56$ and $x + y = 0$. Identify the region enclosed by these two curves. Determine whether to integrate with respect to x or y, and find the area of the region.
4. Consider the curves given by $y = \sin x$ and $y = \cos x$. For each of the following problems, you should include a sketch of the region/solid being considered, as well as a labeled representative slice.
   (a) Sketch the region $\mathcal{R}$ bounded by the y-axis, $y = \cos x$, and $y = \sin x$ up to the first positive value of x at which the curves intersect. What is the exact intersection point of the curves? Be sure to list the *point*, not just an x- or y-value.
   (b) Set up a definite integral with differential dx whose value is the exact area of $\mathcal{R}$.
   (c) Find the exact value of the integral you found in (a).
5. Consider the region $\mathcal{R}$ bounded by $y = \sin(x^2)$, $y = 0$, $x = 0$, and $x = \sqrt{\pi}$. Graph this region in Desmos, sketch the region on your paper, and then set up (but do not evaluate) an integral that represents the exact area of $\mathcal{R}$.
6. Consider the region $\mathcal{S}$ bounded by $y = \sin(x^2)$, $y = 1$, and $x = 0$. Sketch this region in Desmos, and then set up (but do not evaluate) an integral that represents the exact area of $\mathcal{S}$.
7. Trigonometry is based on the unit circle. If trigonometry were based on the unit hyperbola, we would get a different topic called hyperbolic trigonometry. The standard functions in hyperbolic trigonometry are hyperbolic sine, $\sinh(x)$, and hyperbolic cosine, $\cosh(x)$. It turns out, the definitions for these functions are
$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$
$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$
Using these definitions, find the area enclosed by $\sinh(x)$ and $\cosh(x)$ between $x = -1$ and $x = 1$.