MTH 251Z Lab
L'Hôpital's Rule

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Prompts

1. Evaluate the limit.
   (a) $\displaystyle\lim_{x\to 9} \frac{\ln \frac{x}{9}}{81 - x^2}$
   (b) $\displaystyle\lim_{y\to 1} \frac{5^y - 4^y - 1}{y^2 - 1}$
   (c) $\displaystyle\lim_{\theta\to 0} \frac{1 - \cos 3\theta}{1 - \cos 2\theta}$
   (d) $\displaystyle\lim_{z\to \infty} \frac{37z^3}{e^{42z}}$
   (e) $\displaystyle\lim_{t\to \frac{\pi}{2}^+} \frac{\cos t}{1 - \sin t}$
   (f) $\displaystyle\lim_{a\to 0} \frac{e^a - 1 - a}{a^2}$

2. To identify a horizontal asymptote, we compute a limit as $x \to \infty$ or $x \to -\infty$.
   Let $f(t) = \dfrac{3t^3 + 24}{2t^3 + 4t^2 - 5t - 10}$. Compute the following limits to find the horizontal asymptote(s) of f.
   (a) Evaluate $\displaystyle\lim_{t\to\infty} f(t)$
   (b) Evaluate $\displaystyle\lim_{t\to-\infty} f(t)$

3. Computer scientists and chaos theorists often compare functions and their growth rates. Some functions grow at a seemingly similar rate while others grow at very different rates. When one function grows way faster than another, we say it *dominates* the slower function. Mathematically, we say that g **dominates** f if $\displaystyle\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty$ and either $\displaystyle\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$ or $\displaystyle\lim_{x\to\infty} \frac{g(x)}{f(x)} = \infty$.
   (a) Show that e^x dominates x^2.
   (b) Show that x^2 dominates $\sqrt{x}$.
   (c) Show that $\sqrt{x}$ dominates $\ln x$.