

MTH 252Z Lab

The Definite Integral

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Prompts

1. Geometrically, what does $\int_a^b f(x) dx$ represent?
2. What is the formula for the average value of f on the interval $[a, b]$?
3. Let $f(x) = \sqrt{64 - x^2}$.
 - (a) Graph f either by hand or in Desmos.
 - (b) Evaluate $\int_0^8 \sqrt{64 - x^2} dx$.
 - (c) Find the average value of f on $[0, 8]$.
 - (d) Evaluate $\int_{-8}^8 \sqrt{64 - x^2} dx$.
 - (e) Find the average value of f on $[-8, 8]$.
 - (f) Evaluate $\int_{-8}^0 \sqrt{64 - x^2} dx$.
 - (g) Find the average value of f on $[-8, 0]$.
4. Let $f(x) = |5 - x|$ and $g(x) = 2x - 2$.
 - (a) Evaluate $\int_0^8 f(x) dx$.
 - (b) Find the average value of f on $[0, 8]$.
 - (c) Evaluate $\int_{-2}^4 g(x) dx$.
 - (d) Find the average value of g on $[-2, 4]$.
5. Let $f(x) = 2x^2 + 3x - 4$.
 - (a) Write a definite integral to represent the area between the x -axis and the graph of $y = f(x)$ between $x = -2$ and $x = 1$.
 - (b) Write the integral you found in (a) as the limit of a Riemann sum.
 - (c) Write an expression for Δx (the only letter in this expression should be n).
 - (d) Choosing sample points to be right endpoints, we have that $x_i^* = x_i$. Write a formula for x_i for this particular Riemann sum (the only letters in this expressions should be i and n).
 - (e) Evaluate the limit of this Riemann sum by first substituting the expressions you found in (c) and (d) into (b), then expanding the summand, and lastly using your summation and limit properties.