MTH 251Z Lab
Introduction to Calculus

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Prompts

1. Find an equation of the secant line (that is, the line through the two points on the curve) on $y = x^2$ through the points corresponding to $x = 1$ and $x = 3$. Format your conclusion in either point-slope form or slope-intercept form.

2. Galileo dropped a rock from the Leaning Tower of Pisa, about 55m above ground. The distance traveled by the rock t seconds after it left Galileo's hand is $s(t) = 4.9t^2$ meters.

   (a) Does the rock have a constant velocity or does the velocity change? Justify your conclusion.

   (b) How far does the rock travel in 3 seconds? *Don't forget units!*

   (c) Compute the rock's average velocity over the time interval $[2.5, 3]$. *Do not round.*

   (d) Find the rock's average velocity over the given time intervals.

Interval	Average Velocity
$[2.5, 3]$	
$[2.9, 3]$	
$[2.99, 3]$	
$[2.999, 3]$	

   (e) Using the previous table, estimate the *instantaneous velocity* of the rock after 3 seconds.

3. Let $f(x) = \ln x$. The point $P(1, 0)$ is on the graph of $y = f(x)$.

   (a) Use Desmos to graph $y = f(x)$ and plot P.

   (b) Plot the point $R(e, 1)$. If a secant line is drawn from P to R and has slope m_{PR}, then what is m_{PR}?

   (c) Let $Q(x, f(x))$ be a point on the curve $y = f(x)$ that changes as x changes. If m_{PQ} is the slope of the secant line on PQ, then m_{PQ} changes as x changes. Compute the values of m_{PQ} for the different values of x below (just like in class). Round any values to the nearest hundred-thousandth.
   Hint: $m_{PQ} = \dfrac{f(x) - f(1)}{x - 1}$.

x	m_{PQ}
2	
1.5	
1.1	
1.01	
1.001	
1.0001	

(d) Using the previous table, estimate the *slope of the tangent line* to $y = f(x)$ at P.

4. Use the strategy presented in class to find an equation of the tangent line to $y = x^2$ at the point $(2, 4)$. Format your conclusion in either point-slope form or slope-intercept form.

5. For any function $f(x)$, the *difference quotient* we will refer to in this class is $\dfrac{f(x+h) - f(x)}{h}$. For each of the following functions, find its difference quotient and simplify if possible.

   (a) $c(x) = x^3$

   (b) $r(x) = \dfrac{1}{x}$