MTH 254 Final Review

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1. Evaluate the limit.
$$\lim_{(x,y)\to(2,2)} \frac{e^x + \ln(y-1)\cos(\pi xy)}{\sqrt[5]{x^2y^3} + xy}$$

2. Suppose f is a differentiable function of two variables.
   (a) Explain as specifically as possible what $f_y(a,b) = 1$ means.
   (b) Explain as specifically as possible what $f_x(1,1) = -3$ means.
   (c) Explain as specifically as possible what $D_{\mathbf{i}} f(a,b) = 2$ means.
   (d) Explain as specifically as possible what $D_{\mathbf{u}} f(1,1) = -3$ means. For this to be well-defined, what property must $\mathbf{u}$ have?

3. Below are some statements. Determine if the statement is True or False. If the statement is True, you need only write "True" and do not need to provide a justification. If the statement is False, write "False" and justify your answer as specifically as possible.
   (a) Suppose f is a differentiable two-variable function. Then $D_{\mathbf{i}} f(x,y) = f_x(x,y)$.
   (b) Suppose f is a differentiable three-variable function. Then $\dfrac{\partial^3 f}{\partial x \partial y \partial z} = f_{xyz}$.
   (c) Suppose f is a differentiable two-variable function. Then $(a,b) \in \mathbb{R}^2$, $f_{xy}(a,b) = f_{yx}(a,b)$.
   (d) For all (a,b) in the domain of a two-variable function f, $\lim\limits_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.
   (e) $\displaystyle\int_{-1}^{2}\int_{0}^{6} x^2 \sin(x-y)\ dx\ dy = \int_{0}^{6}\int_{-1}^{2} x^2 \sin(x-y)\ dy\ dx.$

4. Let $z = f(x,y)$, where $f(x,y) = 1 + x + y^2 - \sin(xy) + \ln(x^4 + y^2)$, and let S be the surfaced produced by that equation.
   (a) Find f_{xx}.
   (b) Find f_{xy}.
   (c) Find f_{yyx}.
   (d) Find the linear approximation to S at the point $(1,0,2)$.

5. Find the linear equation of the plane tangent to the surface whose equation is $z = e^x \cos y$ at $(0,0,1)$.

6. Find the symmetric equations for the normal line to the surface $\sin(xyz) = x + 2y + 3z$ at the point $(2,-1,0)$.

7. If $\sin(xyz) = x + 2y + 3z$, find $\dfrac{\partial x}{\partial y}$.

8. Let $u = x^2y^3 + z^4$, with $x = p + 3p^2$, $y = pe^p$, and $z = p\sin p$. Find $\dfrac{du}{dp}$.

9. Let $v = x^2 \sin y + ye^{xy}$, with $x = s + 2t$, and $y = st$. Find $\dfrac{\partial v}{\partial s}$ when $s = 0$ and $t = 1$.

10. Let $F(x, y, z) = x^2 e^{yz^2}$, S be the level surface whose equation is $F(x, y, z) = 4$, and let P be the point $(2, 0, 4)$ on S.
    (a) Find ∇F.
    (b) Find a vector pointing in the direction for which $D_{\mathbf{u}}F(2, 0, 4)$ maximal.
    (c) Find the maximal value of $D_{\mathbf{u}}F(2, 0, 4)$.
    (d) Find an equation for the tangent plane to S at P.
    (e) Find a vector equation for the normal line to S at P.

11. Let $f(x, y) = 3xy - x^2y - xy^2$. Find the local extrema and saddle points of f. Show all of your work to justify your conclusion.

12. Use Lagrange multipliers to find the absolute extrema of $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$ subject to the constraint $x^2 + y^2 = 4$.

13. Find the absolute extrema of $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$ on $\mathscr{D} = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4\}$.

14. Use a Riemann sum with $m = 3$ and $n = 2$ to estimate $\displaystyle\iint_R x^2 \sin(x - y)\, dA$ where $R = [-1, 2] \times [0, 6]$. Take the sample points to be upper-left corners. Round your conclusion to the nearest thousandth.

15. Find the exact value of $\displaystyle\iint_R x^2 \sin(x - y)\, dA$ where $R = [-1, 2] \times [0, 6]$. Then approximate this value to the nearest thousandth.

16. Let V be the volume of the solid beneath the surface $y + 2xe^y - z = 0$ and above the rectangle $R = [1, 2] \times [0, 2]$.
    (a) Set up an integral that represents V.
    (b) Find the exact value of V.

17. Convert $(1, -2, 7)$ to cylindrical coordinates. Draw a three-dimensional coordinate system, and plot your point. Round your values to the nearest hundredth.

18. Convert $(-3, -1, 2)$ to spherical coordinates. Draw a three-dimensional coordinate system, and plot your point. Round your values to the nearest hundredth.

19. Sketch the solid described by $\rho \leq 1$, $0 \leq \varphi \leq \frac{\pi}{6}$, and $0 \leq \theta \leq \pi$.

20. Evaluate $\displaystyle\iiint_E \sin y\, dV$, where E lies below the plane $z = x$ and above the triangular region whose vertices are $(0, 0, 0)$, $(\pi, 0, 0)$ and $(0, \pi, 0)$.

21. Evaluate $\displaystyle\iiint_E \sqrt{x^2 + y^2 + z^2}\, dV$, where E is the solid above the cone $z = \sqrt{x^2 + y^2}$ and between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.