MTH 254 Midterm Review

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1. Let $P(3,-1,-4)$, $Q(2,5,4)$, $R(-2,7,1)$, $\mathbf{u} = \overrightarrow{PQ}, \mathbf{v} = \overrightarrow{PR}$.
   (a) Draw a 3-dimensional rectangular coordinate system. Label the positive sides of the axes according to the right-hand rule, draw tick marks, and provide a scale.
   (b) Plot P, Q, and R. Be sure to include any guiding lines to give context to the points.
   (c) Graph $\mathbf{u}$ and $\mathbf{v}$.
   (d) Write $\mathbf{u}$ in component form.
   (e) Write $\mathbf{v}$ in terms of $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$.
   (f) Find $\mathbf{u} + \mathbf{v}$.
   (g) Find $\mathbf{u} - \mathbf{v}$.
   (h) Find $2\mathbf{u} + 3\mathbf{v}$.
2. Let $A(1,1)$, $B(2,3)$, $C(5,4)$, $\mathbf{u} = \overrightarrow{AB}, \mathbf{v} = \overrightarrow{BC}$.
   (a) Draw a Cartesian plane. Label the positive sides of each axis, draw tick marks, and provide a scale.
   (b) Plot A, B, C.
   (c) Graph the position vectors for $\mathbf{u}$ and $\mathbf{v}$.
   (d) Graph $\mathbf{u} + \mathbf{v}$. Label the vector.
   (e) Graph $\mathbf{u} - \mathbf{v}$. Label the vector.
   (f) Graph $\text{proj}_{\mathbf{v}}\,\mathbf{u}$. Label the vector.
   (g) Graph $\text{proj}_{\mathbf{u}}\,\mathbf{v}$. Label the vector.
3. Let C be the curve determined by the vector function $\mathbf{r}(t) = \langle 2t, t^2, \frac{1}{3}t^3 \rangle$ with $2 \leq t \leq 4$. Find the exact length of C.

4. Let $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ and $\mathbf{v} = \langle -3, 1, 4 \rangle$.
   (a) Draw a 3-dimensional rectangular coordinate system. Label the positive sides of the axes according to the right-hand rule, draw tick marks, and provide a scale. Graph both $\mathbf{u}$ and $\mathbf{v}$ on your coordinate system.
   (b) Find $|\mathbf{u}|$.
   (c) Find a unit vector in the same direction as $\mathbf{u}$.
   (d) Find the smallest angle between $\mathbf{u}$ and $\mathbf{v}$. Round your conclusion to the nearest tenth of a radian.
   (e) Are $\mathbf{u}$ and $\mathbf{v}$ orthogonal?
   (f) Find a nonzero vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.
   (g) Find the area of the parallelogram formed by $\mathbf{u}$ and $\mathbf{v}$.
   (h) Find $\text{proj}_{\mathbf{u}}\, \mathbf{v}$.
   (i) Find $\text{comp}_{\mathbf{v}}\, \mathbf{u}$.
   (j) Find the symmetric equations for the line passing through the terminal points of $\mathbf{u}$ and $\mathbf{v}$.
   (k) Find the parametric equations for the line through the terminal point of the position vector for $\mathbf{u}$ with direction vector $\mathbf{v}$.
   (l) Find a linear equation of the plane containing $\mathbf{u}, \mathbf{v}$, and $\mathbf{0}$.

5. Let $\mathbf{r}(t) = \langle t^3 - 4t, \sin\left(\frac{\pi}{3}t\right)\rangle$ with $-3 \le t \le 3$. Let C be the curve determined by $\mathbf{r}$.
   (a) Produce a table of values to find the points on C.
   (b) Draw a Cartesian plane, and sketch C. Include arrows to indicate the direction that $\mathbf{r}$ travels as t increases.
   (c) Graph $\mathbf{r}(1)$. Label the vector.
   (d) Graph $\mathbf{T}(1)$. Label the vector.
   (e) Graph $\mathbf{N}(1)$. Label the vector.
   (f) Find $\mathbf{T}(1)$.
   (g) Find an equation of the tangent line to C at the point where $t = 1$.
   (h) Find the curvature of C when $t = 1$. Round your conclusion to the nearest hundredth.

6. Suppose a particle is moving in space with initial position $\mathbf{r}(0) = \mathbf{i} + \mathbf{k}$ and velocity

$$\mathbf{v}(t) = \left\langle \frac{2}{1+t^2}, 5e^{5t-5}, \frac{4}{t+1} \right\rangle$$

   where $\mathbf{v}(t)$ is measured in meters per second. Let C represent the path the particle takes as t increases. Find the following quantities. Use exact values, and respond to each part with a sentence with units.
   (a) The velocity of the particle after 1 second.
   (b) The position of the particle at time t.
   (c) The position of the particle after 1 second.
   (d) The acceleration of the particle at time t.
   (e) The acceleration of the particle after 1 second.
   (f) The exact displacement vector of the particle in the first 3 seconds.
   (g) The tangential component of acceleration after 1 second.
   (h) The normal component of acceleration after 1 second.
   (i) The curvature of C at the point when $t = 1$.