Portfolio

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Purpose

Throughout this course, you will work through a few prompts and collect them together in a course portfolio. These prompts represent key topics in this course. As you progress through the course, you will add to this portfolio and also have an opportunity to go back and edit any previous responses. By the end of the term, you will have a portfolio of work that demonstrates your understanding of this course.

Assignment

This portfolio will be worth 100 points throughout the term. Each prompt will be given at the beginning of the course, but you will not be able to complete them until you progress through certain topics in the course. For any individual prompt, you can submit it for corrections and critiques, and you will receive feedback to assist you in editing your response.

Your response to each prompt should adhere to these guidelines:

- Each prompt will begin on a new page.
- The instructions for the prompt are written before any work is shown.
- Any computations are fully worked out without the aid of a calculator or computer.
- Any work that is unclear is justified with full sentences.
- The conclusion is written as the end of the response.
- Any values are given exactly, unless rounding is specifically asked for.
- Proper notation is always given.

Prompts

Prompt 1. Let $\mathbf{u} = \langle 1, 3, 5 \rangle$ and $\mathbf{v} = -4\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ be vectors in $\mathbb{R}^3$. Find the following.

a. $\mathbf{u} + \mathbf{v}$.

b. $-3\mathbf{u}$.

c. $|\mathbf{u}|$.

d. The unit vector in the same direction as $\mathbf{u}$.

e. $\mathbf{u} \cdot \mathbf{v}$.

f. The angle between $\mathbf{u}$ and $\mathbf{v}$.

g. $\text{proj}_{\mathbf{v}}\, \mathbf{u}$.

Prompt 2. Let $P(1, 3, 5)$ and $Q(-4, 3, -7)$ be points in $\mathbb{R}^3$.

a. Find an equation of the line through P and Q.

b. Find an equation for the plane on the point P, Q, and the origin.

Prompt 3. Two particles travel along the curves whose vector equations are listed below. Do the particles collide?

$$\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle \qquad \text{and} \qquad \mathbf{r}_2(t) = \langle 1 + 2t, 2 + t, 1 + 14t \rangle$$

Prompt 4. A particle is moving in space with position function $\mathbf{r}(t) = e^t\mathbf{i} + te^t\mathbf{j} + te^{t^2}\mathbf{k}$, measured in meters after t seconds. Find the following quantities, exactly.

a. The acceleration of the particle after 1 second.

b. The tangential component of acceleration after 1 second.

c. The normal component of acceleration after 1 second.

Prompt 5. Let $f(x,y) = y - \arctan x$.

a. Evaluate $f\left(2, \frac{\pi}{4}\right)$.

b. Draw a contour map of the function $f(x,y) = y - \arctan x$ showing the level curves corresponding to $k = -2, 0, 2, 4$.

Prompt 6. Let $f(x,y) = \sin(2x+3y)$. Let S be the surface produced by $z = f(x,y)$ and P be the point $\left(\frac{\pi}{16}, \frac{\pi}{24}, \frac{\sqrt{2}}{2}\right)$.

a. Find f_{yxy}.

b. Find an equation of the tangent plane to S at P.

c. Find a vector equation for the normal line to S at P.

Prompt 7. Let $f(x,y) = e^{-x^2-y^2}(x^2+2y^2)$.

a. Find the gradient vector $\nabla f(x,y)$.

b. Find an equation for the tangent plane to the level surface $z = f(x,y)$ at the point $\left(1, 1, \frac{3}{e^2}\right)$.

Prompt 8. Let R be the rectangle $[1,9]\times[2,4]$, and let $f(x,y) = \dfrac{\sqrt{x}}{y^2}$. Evaluate $\displaystyle\iint_R f(x,y)\,dA$.