Linear Algebra and Circuitry

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Introduction: Who are we

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Circuits are involved in our everyday lives. By looking at electrical circuits and their relationship with linear algebra, we will see how Gaussian elimination is used for solving systems of equations involving voltage, resistance, and current.

- Simplify the process instead of computing a larger set of equations that involve many unknown variables that can be easy to miscalculate and harder to detect an error.
- Focus on the methods on and how one can simplify using an augmented matrix and elementary row operations to get the same results for an unknown voltage

Introduction Cont.

One of the main components of Electrical Engineering is identifying the unknown voltage, current, or resistance in a circuit. Circuits range from series to parallel, and often can be extensive in finding missing information. For this presentation's purpose, we will primarily focus on direct current (DC).

Series and Parallel Circuits

There are two main types of circuits in electrical engineering.

- Parallel Circuit: All elements of the circuit to have their own nodal points. In other words, two or more paths are needed for current to flow through. Wiring in your house and your cell phone are great examples for this.
- Series Circuit: Only one path is allowed for current to flow through a circuit. Christmas lights are an example of this.

Circuits: Ohm's Law

- In order to find missing values such as Voltage (V), Current (I) or Resistances (R), we use Ohm's Law.
- Ohm's Law states that "current flowing in an electric circuit is directly proportional to the applied voltage and inversely proportional to the resistance of the material."[**?**]
- In other words, this can be considered a pretty simple way to find whatever part of an electrical circuit you are looking for. The Law states that $V = IR$.

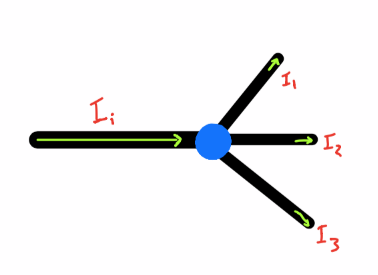
Circuits Cont.: Kirchoff's Law

Kirchoff's Law consists of two statements that signify important processes for understanding current and voltage activity.

- Kirchoff's 1st Law: Current flowing into a node must be equal to current flowing out of it.
- Kirchoff's 2nd Law: The sum of all voltages around any closed loop in a circuit must equal zero. This is a consequence of charge conservation and conservation of energy.

To simplify our process of solving unknown variables, we use linear algebra to solve our system of equations.

Kirchoff's First Law

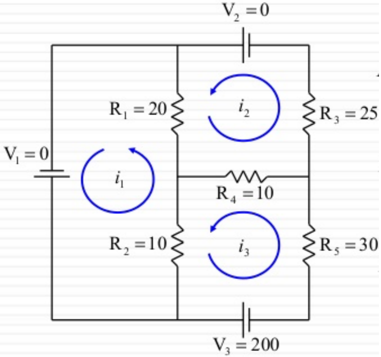


$I_i = I_1 + I_2 + I_3$
All the current flowing through the Blue point "The Node" will equal I_i

Nodal Voltage Analysis

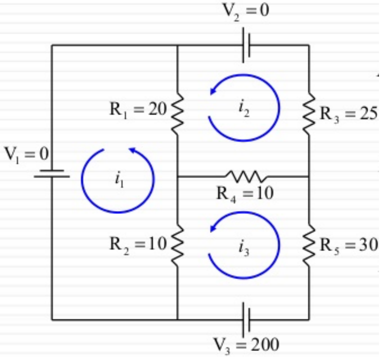
To solve certain systems of equations in circuit problems, Nodal Voltage Analysis lets us see multiple resistances, voltages, and currents. Usually this involves many steps requiring substitution and elimination. Therefore, linear algebra helps us solve these systems of equations in a more simpler fashion.

How to Apply Linear Algebra to a Circuit Diagram



Example: Find the current travelling through the system...[**?**]

How to Apply Linear Algebra to a Circuit Diagram



First step: Setup the equation:

- Left flow loop:
  $20(i_1 - i_2) + 10(i_1 - i_3) = 0$
- Upper right loop flow :
  $25i_2 + 10(i_2 - i_3) + 20(i_2 - i_1) = 0$
- Bottom right flow loop :
  $30i_3 + 10(i_3 - i_2) + 10(i_3 - i_1) = 200$

How to Apply Linear Algebra to a Circuit Diagram Continued

Next, convert the systems of equation created into a Matrix.

$$20(i_1 - i_2) + 10(i_1 - i_3) = 0$$
$$25i_2 + 10(i_2 - i_3) + 20(i_2 - i_1) = 0$$
$$30i_3 + 10(i_3 - i_2) + 10(i_3 - i_1) = 200$$

Equation in matrix form:

$$\begin{bmatrix} 30 & -20 & -10 & 0 \\ -20 & 55 & -10 & 0 \\ -10 & -10 & 50 & 200 \end{bmatrix}$$

How to Apply Linear Algebra to a Circuit Diagram

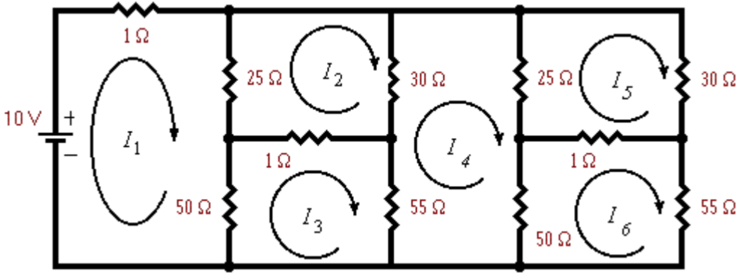
Row Reduce A until the matrix is in RREF $\begin{bmatrix} 30 & -20 & -10 & 0 \\ -20 & 55 & -10 & 0 \\ -10 & -10 & 50 & 200 \end{bmatrix}$

After Row reducing the matrix A you get: $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

How to Apply Linear Algebra to a Circuit Diagram

A in RREF: $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

$$\begin{cases} i_1 = 3 \\ i_2 = 2 \\ i_3 = 5 \end{cases}$$


1 Ω
10 V
+
−
I_1
50 Ω
25 Ω
I_2
1 Ω
I_3
30 Ω
I_4
55 Ω
25 Ω
I_5
30 Ω
1 Ω
50 Ω
I_6
55 Ω

Thinking Bigger

1. $i_1 + 25(i_1 - i_2) + 50(i_1 - i_3) = 10$
2. $25(i_2 - i_1) + 30(i_2 - i_4) + 1(i_2 - i_3) = 0$
3. $50(i_3 - i_1) + 1(i_3 - i_2) + 55(i_3 - i_4) = 0$
4. $55(i_4 - i_3) + 30(i_4 - i_2) + 25(i_4 - i_5) + 50(i_4 - i_6) = 0$
5. $25(i_5 - i_4) + 30i_5 + 1(i_5 - i_6) = 0$
6. $50(i_6 - i_4) + 1(i_6 - i_5) + 55i_6 = 0$

$$\begin{bmatrix} 76 & -25 & -50 & 0 & 0 & 0 & 10 \\ -25 & 56 & -1 & -30 & 0 & 0 & 0 \\ -50 & -1 & 106 & -55 & 0 & 0 & 0 \\ -0 & -30 & -55 & 160 & -25 & -50 & 0 \\ 0 & 0 & 0 & -25 & 56 & -1 & 0 \\ 0 & 0 & 0 & -50 & -1 & 106 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & .478 \\ 0 & 1 & 0 & 0 & 0 & 0 & .348 \\ 0 & 0 & 1 & 0 & 0 & 0 & .353 \\ 0 & 0 & 0 & 1 & 0 & 0 & .239 \\ 0 & 0 & 0 & 0 & 1 & 0 & .109 \\ 0 & 0 & 0 & 0 & 0 & 1 & .114 \end{bmatrix}$$

Conclusion

As a result, linear algebra can be used to solve systems of equations known as Nodal Voltage Analysis to help us find missing currents, voltages, or resistances in an electrical circuit problem. By using row elementary operations, we have demonstrated that setting up our systems of equations is a lot smoother and simpler when set up in an augmented matrix.

Bibliography

Thanks and Questions

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