MTH 261 Midterm Exam Sample

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Name: ____________________

For instructor use only
Score : /125

Some notes from Damien about the exam:

- You will have 2 hours 20 minutes to complete this exam. When you are finished, please turn it in to Damien.
- You may use a graphing calculator, but once you begin the exam, you *may not* use an internet sources or any other external source for assistance. This includes your notes, peers, textbook, tutors, other human beings, robots, or any other source I may have failed to mention.
- You may ask Damien for clarification on any problems, but you *may not* converse with anyone else regarding this exam.
- If you have any questions, ask Damien!
- The use of scratch paper is encouraged and should be included at the end of the exam. Please clearly label all work for each prompt.
- Please read all of the instructions carefully.
- Please show all of your work to each prompt. This is exceedingly important!
- Please use exact values unless otherwise stated.
- Please use vector notation any time you are writing a vector.
- Please show all row reduction steps.
- May the Force be with you!

Furthermore, please read all of the conditions below before beginning the exam. Indicate that you have read and understand these conditions by placing your initials on the line below.

- I have and will follow all of the guidelines stated above.
- I have not and will not cheat or violate any part of the Academic Integrity Policies.
- I will not provide or distribute any portion of this document to anyone other than the instructor.
- All work attached herein is my own authentic work.

Initials : ______________

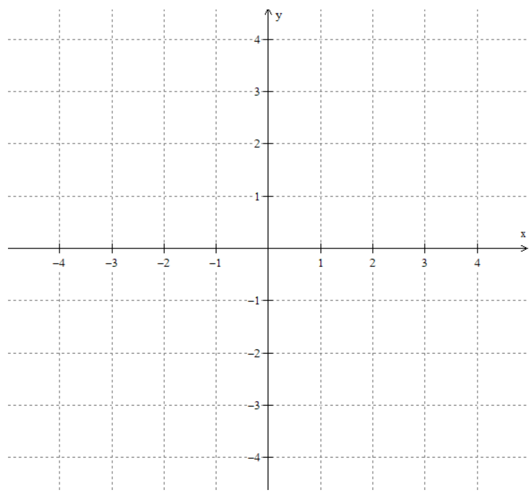
(8) 1. Suppose A, B, and C are matrices with the following sizes: A is $42 \times \alpha$, B is $37 \times \beta$, and C is $\gamma \times \delta$ for some $\alpha, \beta, \gamma, \delta \in \mathbf{N}$.

(a) Is it possible that $C = BA$? If so, then what must β be? If not, please explain why as specifically as you can.

(b) Is it possible that $C = B + A$? If so, then what must β be? If not, please explain why as specifically as you can.

(9) 2. Let $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \in \mathbb{R}^2$. On the provided set of axes, graph the following vectors as arrow vectors beginning at $\mathbf{0}$. Clearly label each vector on the graph.

(a) $\mathbf{a} = -2\mathbf{u}$ (b) $\mathbf{b} = \mathbf{u} + \mathbf{v}$ (c) $\mathbf{c} = \mathbf{u} - \mathbf{v}$



(8) 3. Determine if the statement is True or False. If the statement is True, you need only write "True" and do not need to provide a justification (though one may provide partial credit). If the statement is False, write "False" and justify your conclusion as specifically as possible. (Do not write "T" or "F"; please write the full word)

(a) The matrix equation $A\mathbf{x} = \mathbf{b}$ always has the unique solution $\mathbf{x} = A^{-1}\mathbf{b}$.

(b) If A, B, C are $n \times n$ matrices, then $(ABC)^T = A^T B^T C^T$.

(10) 4. Solve the system of equations using Gaussian Elimination. Provide your conclusion in parametric vector form.

$$\begin{cases} x_1 + 2x_2 = 1 \\ x_1 + 2x_2 - x_3 = 2 \\ 2x_1 + 4x_2 + 2x_3 = 0 \end{cases}$$

(8) 5. Let $S = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$, where

$$\mathbf{u} = \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} \quad , \quad \mathbf{v} = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} \quad , \quad \mathbf{w} = \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix}$$

Determine if S is linearly independent or not. If the set is linearly dependent, then find a linear dependence relation. If the set is linearly independent, justify your conclusion as specifically as possible.

6. Let $\mathbf{u}, \mathbf{v}, \mathbf{b} \in \mathbb{R}^3$ such that

$$\mathbf{u} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} \quad , \quad \mathbf{v} = \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix} \quad , \quad \mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ -6 \end{bmatrix}$$

(5) (a) Determine if $\mathbf{b} \in \operatorname{span}\{\mathbf{u}, \mathbf{v}\}$ or not. Show all work that leads to your conclusion.

(5) (b) Determine if $\{\mathbf{u}, \mathbf{v}, \mathbf{b}\}$ is linearly independent or not. If the set is linearly dependent, then find a linear dependence relation. If the set is linearly independent, justify your conclusion as specifically as possible.

(15) 7. Evaluate the matrix expression or explain why the expression is undefined. Show all of your work to support your conclusion.

(a) $3\begin{bmatrix}1 & 1\\ 2 & -1\end{bmatrix}^T - \begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ -1 & 1\\ 1 & 0\end{bmatrix}$

(b) $2I_4 + \begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\\ 0 & 1\\ 1 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}3 & 1 & 4\\ 1 & 5 & 9\\ 2 & 6 & 5\\ 3 & 5 & 8\\ 9 & 7 & 9\end{bmatrix}$

8. The following are Fibonacci matrices. Determine if A is invertible or singular. If A is invertible, find A^{-1}. If A is singular, justify your conclusion as specifically as possible.

(5) (a) $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

(6) (b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 5 & 8 \end{bmatrix}$

(8) 9. Provide the following definitions as specifically as possible.

(a) Define what it means for a vector $\mathbf{u}$ to be in $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$, where $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p \in \mathbb{R}^n$.

(b) Define what it means for the set $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ to be linearly independent.

(10) Let $A = \begin{bmatrix} 7 & -4 & 32 & 24 \\ -3 & 1 & -13 & -11 \\ 8 & -1 & 33 & 31 \end{bmatrix}$. Use the fact that $\mathrm{RREF}(A) = \begin{bmatrix} 1 & 0 & 4 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ to answer the following questions.

(6) (a) Find the general solution to $A\mathbf{x} = \mathbf{0}$, and give your answer in parametric **vector** form.

(4) (b) Do the columns of A span $\mathbb{R}^3$? Why or why not? Justify your conclusion as specifically as possible.

(12) 11. A system of equations in the variables x_1 and x_2 is given below, where $\alpha, \beta \in \mathbb{R}$. Determine what values α and β must be that will produce the desired number of solutions in each part.

$$\begin{array}{rcrcl} x_1 & + & 3x_2 & = & \alpha \\ 4x_1 & - & \beta x_2 & = & 8 \end{array}$$

(a) No solutions.

(b) Exactly one solution.

(c) Infinitely many solution.

(6) 12. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects points across the line $x_1 = x_2$ and then rotates them about the origin $\frac{\pi}{2}$ radians. Find the standard matrix for T.

13. Define the transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 + x_2$.

(4) (a) Evaluate $T\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right)$ and $T\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) + T\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right)$.

(4) (b) Evaluate $T\left(c\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right)$ and $cT\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right)$.

(2) (c) Is T a linear transformation? Circle one: Yes No

Your work above in parts (a) and (b) serve as justification, so no additional justification is necessary.