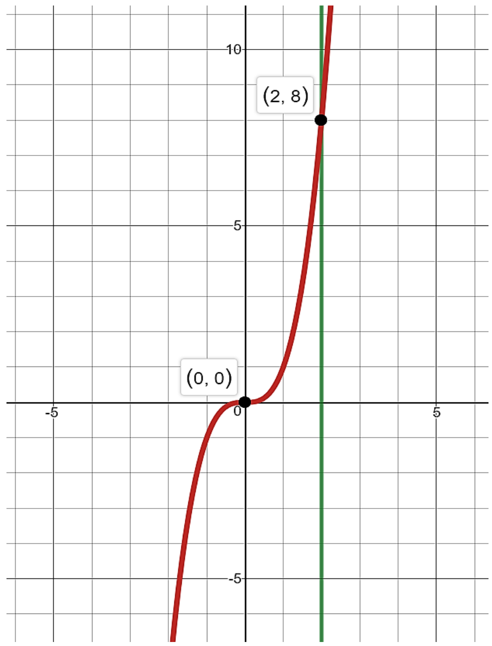
Exam 1 Review / Math 251 / Differential Calculus

1. The point $P(2, 8)$ lies on the curve $y = x^3$

a. Graph the curve $y = x^3$ and plot the point P.
(green line is just to show the point on the graph)



b. If Q is the point (x, x^3), find the slope of the secant line PQ for the following values of x, rounding to five decimal places:

X	$m_{sec} = \frac{f(x) - f(2)}{x - 2}$
1.5	9.25
1.9	11.41
1.99	11.9401
2	Do not calculate at 2
2.01	12.0601
2.1	12.61
2.5	15.25

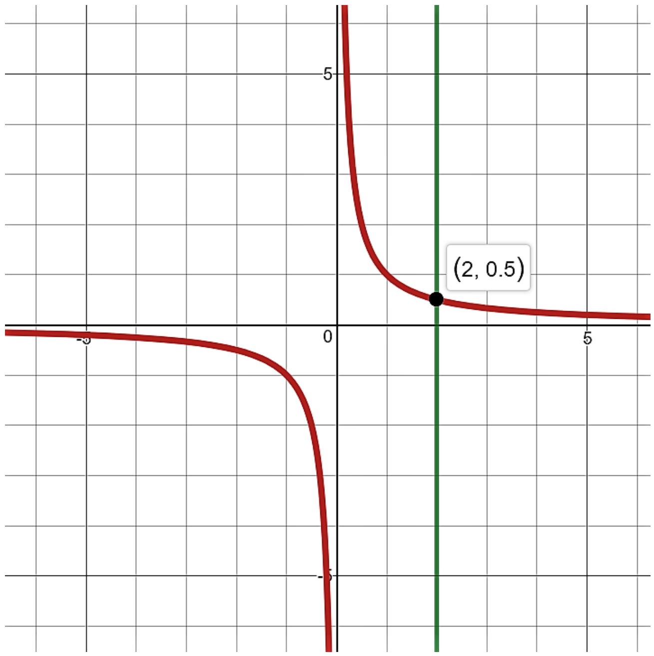
c. Taking part (b) into consideration, make an educated guess at the value of the slope of the tangent line to the curve at $P(2, 8)$:

$$m_{tan} \approx \frac{11.9401 + 12.0601}{2} \approx 12.0001$$

I believe the slope of tangent at point (2, 8) is roughly 12

2. **The point $P(2, \frac{1}{2})$ lies on the curve $y = \frac{1}{x}$**

a. **Graph the curve $y = \frac{1}{x}$ and plot the point P.**
(green line is just to show the point on the graph)



b. **If Q is the point $(x, \frac{1}{x})$, find the slope of the secant line PQ for the following values of x, rounding to five decimal places:**

X	$m_{sec} = \frac{f(x) - f(2)}{x - 2}$
1.5	$-\frac{1}{3}$, $or\ -0.33333$
1.9	$-\frac{5}{19}$, $or\ -0.26316$
1.99	$-\frac{50}{199}$, $or\ -0.25126$
2	Do not calculate at 2
2.01	$-\frac{50}{201}$, $or\ -0.24876$
2.1	$-\frac{5}{21}$, $or\ -0.23810$
2.5	$-\frac{1}{5}$, $or\ -0.2$

c. **Taking part (b) into consideration, make an educated guess at the value of the slope of the tangent line to the curve at $P(2,\frac{1}{2})$:**

$$m_{tan} \approx \frac{-0.25126 + (-0.24876)}{2} \approx -0.25001$$

I believe the slope of tangent at point $(2,\frac{1}{2})$ is roughly -0.25

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3. **Sketch the graph of a function f satisfying all of the given conditions:**

a. $\lim_{x\to-\infty} f(x) = -\infty$

b. $\lim_{x\to\infty} f(x) = \infty$

c. $\lim_{x\to 0^-} f(x) = -\infty$

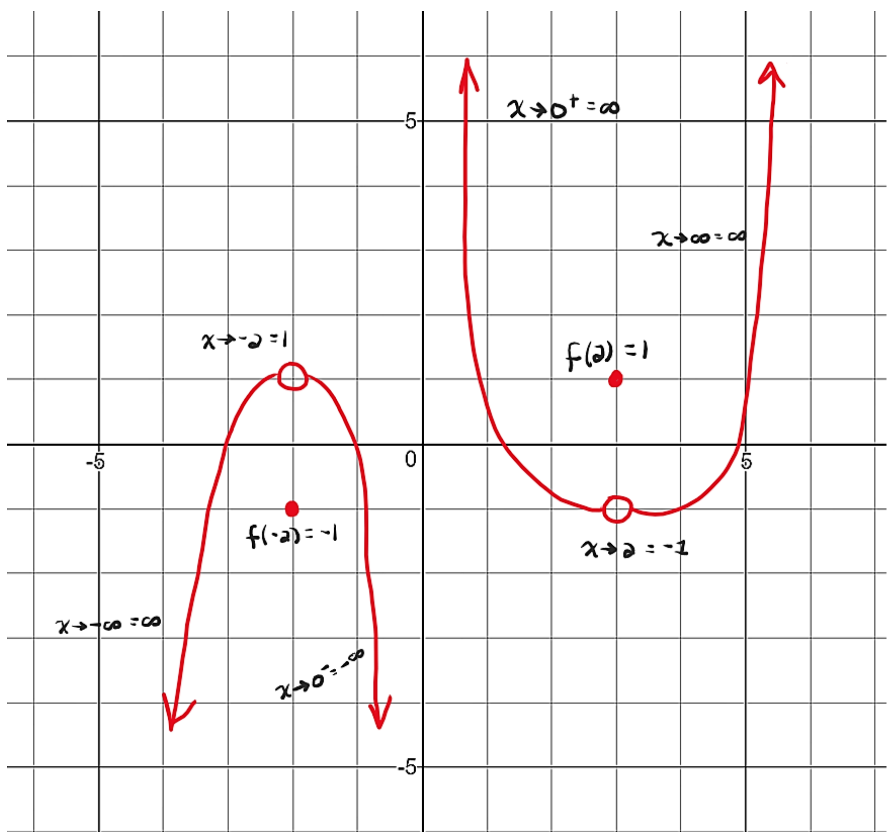
d. $\lim_{x\to 0^+} f(x) = \infty$

e. $\lim_{x\to -2} f(x) = 1$

f. $\lim_{x\to 2} f(x) = -1$

g. $f(-2) = -1$

h. $f(2) = 1$



4. Sketch the graph of a function f satisfying all of the given conditions:

a. $\lim_{x\to-\infty} f(x) = 3$

b. $\lim_{x\to\infty} f(x) = -4$

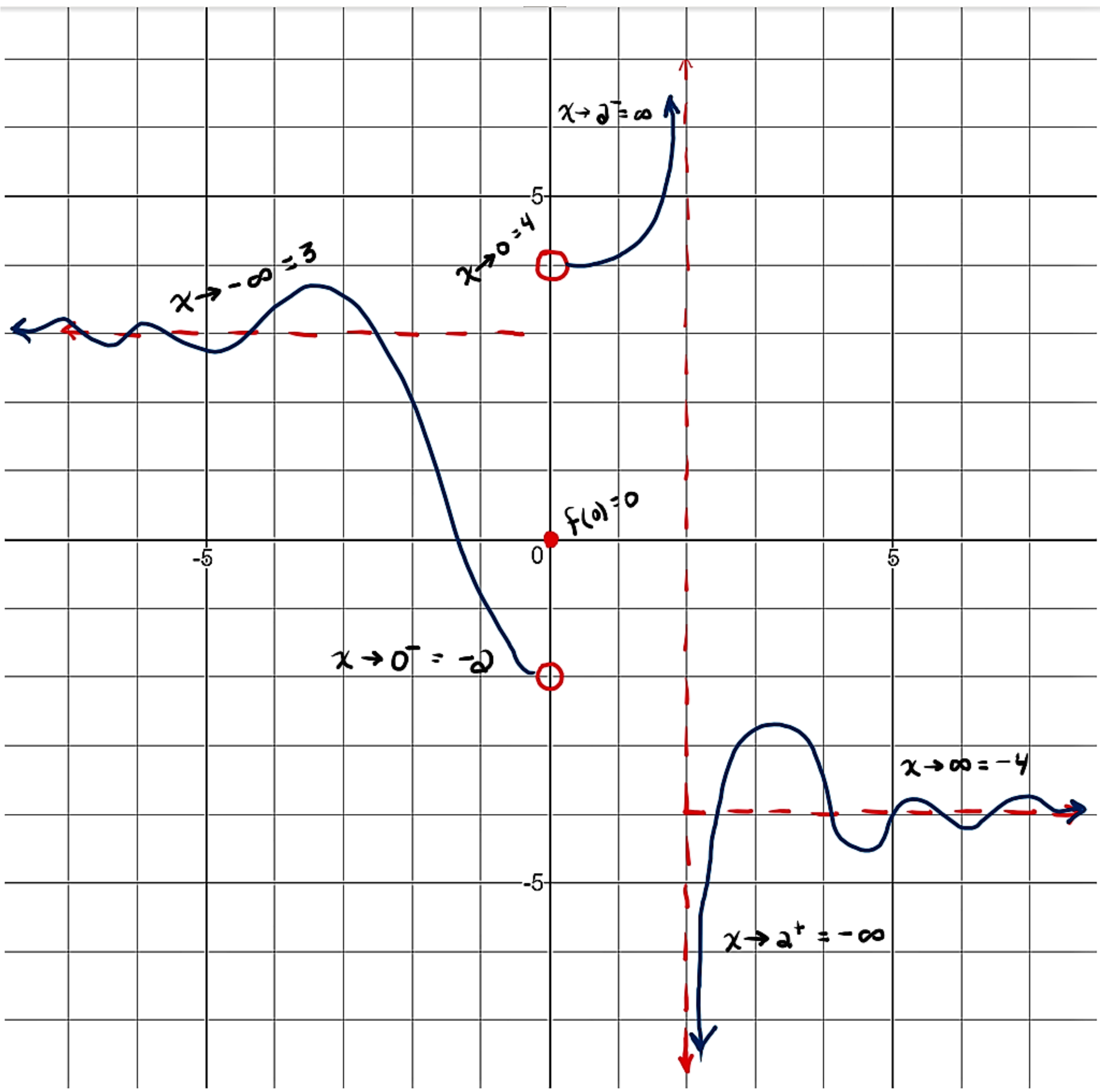
c. $\lim_{x\to 0^-} f(x) = -2$

d. $\lim_{x\to 0^+} f(x) = 4$

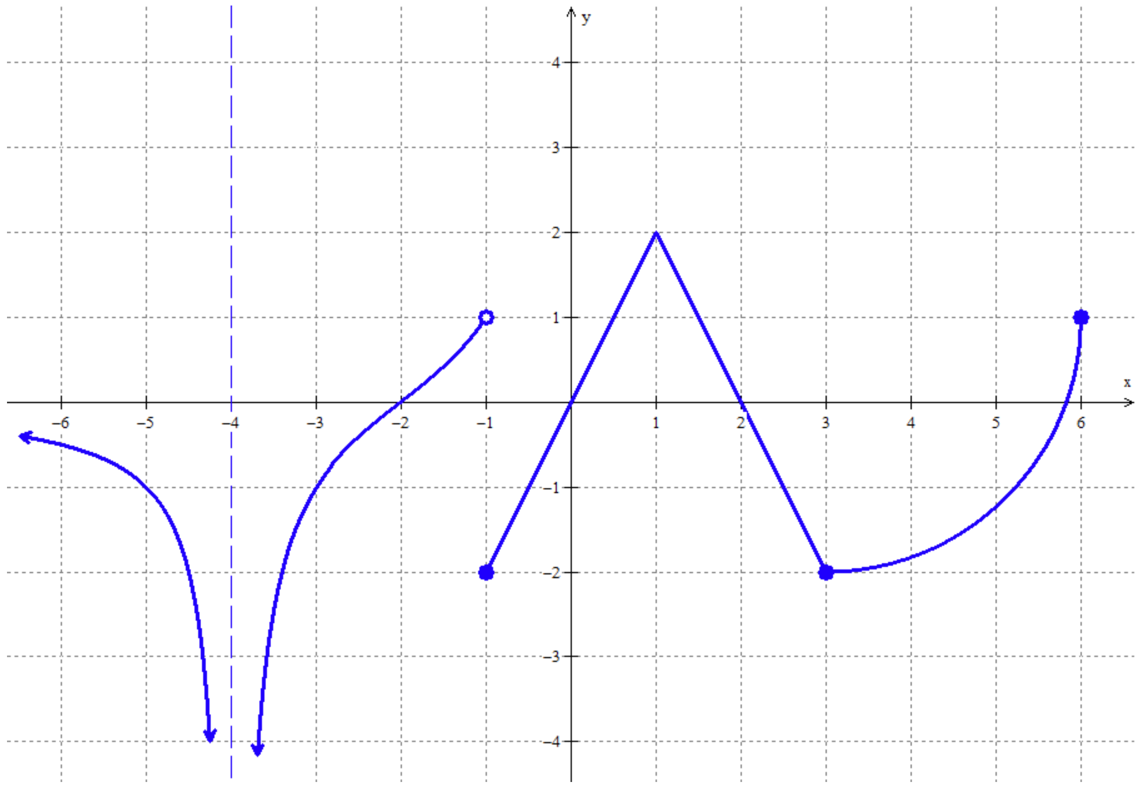
e. $\lim_{x\to 2^-} f(x) = \infty$

f. $\lim_{x\to 2^+} f(x) = -\infty$

g. $f(0) = 0$



5. Using the graph of $y = f(x)$ below, answer the following questions.



a. Is f continuous on $(-\infty, -4)$?
No, if -4 is being included in the interval

b. Is f continuous from the left at $x = -1$?
No

c. Is f continuous from the right at $x = -1$?
Yes

d. What is $\lim_{x \to -4^-} f(x)$?
$-\infty$

e. What is $\lim_{x \to -4^+} f(x)$?
$-\infty$

f. What is $\lim_{x \to -4} f(x)$?
$-\infty$

g. What is $f(-4)$?
Undefined

h. Is f continuous at $x = -4$?
No

i. What is $\lim_{x \to -2^-} f(x)$?
0

j. What is $\lim_{x \to -2^+} f(x)$?
0

k. What is $\lim_{x \to -2} f(x)$?
0

l. What is $f(-2)$?
0

m. Is f continuous at $x = -2$?
Yes

n. What is $\lim_{x \to -1^-} f(x)$?
1

o. What is $\lim_{x \to -1^+} f(x)$?
-2

p. What is $\lim_{x\to -1} f(x)$?
Does Not Exist

q. What is $f(-1)$?
-2

r. Is f continuous at $x = -1$?
No

s. What is $\lim_{x\to 3^-} f(x)$?
-2

t. What is $\lim_{x\to 3^+} f(x)$?
-2

u. What is $\lim_{x\to 3} f(x)$?
-2

v. What is $f(3)$?
-2

w. Is f continuous at $x = 3$?
Yes

x. Where are the discontinuities of f? Identify each discontinuity as either a removable, jump, or infinite discontinuity.
At $x = -4$, Infinite discontinuity,
and $x = -1$ Jump discontinuity

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6. a. $\lim_{x\to 3} \frac{x^2-5x+6}{2x^2-5x-3}$

$= \frac{1}{7}$

b. $\lim_{x\to \infty} \frac{x^2-5x+6}{2x^2-5x-3}$

$= \frac{1}{2}$

c. $\lim_{y\to -1} \frac{y^3+1}{y^2-1}$

Sum of Cubes: $(y+1)(y^2-y+1)$

$= -\frac{3}{2}$

d. $\lim_{y\to \infty} \frac{y^3+1}{y^2-1}$

$= -\infty$

6. **e.** $\lim_{z \to 0} \frac{cos(z)}{z^4 - 1}$

$= -1$

f. $\lim_{t \to \infty} \frac{\sqrt{4t^2+1}}{t-1}$

$= 2$

g. $\lim_{\alpha \to 2^-} \frac{|\alpha-2|}{\alpha^2 - 3\alpha + 2}$

$= \lim_{\alpha \to 2^-} \frac{-(\alpha-2)}{\alpha^2 - 3\alpha + 2}$ ← Limit approaching $-\infty$: $-(x)$

$= -(1)$

h. $\lim_{\alpha \to 2^-} \frac{|\alpha-2|}{\alpha^2 - 3\alpha + 2}$

$= \lim_{\alpha \to 2^-} \frac{+(\alpha-2)}{\alpha^2 - 3\alpha + 2}$ ← Limit approaching ∞: x

$= 1$

i. **Does Not Exist, approaching different values from right & left**

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7. **a.** **True,** pg. 153 Higher Derivatives

b. **False,** the graph of $f(t)$ does not exist for values t < -2
(only continuous approaching -2 from the right)

c. **False,** pg. 113 Continuity

d. **True**

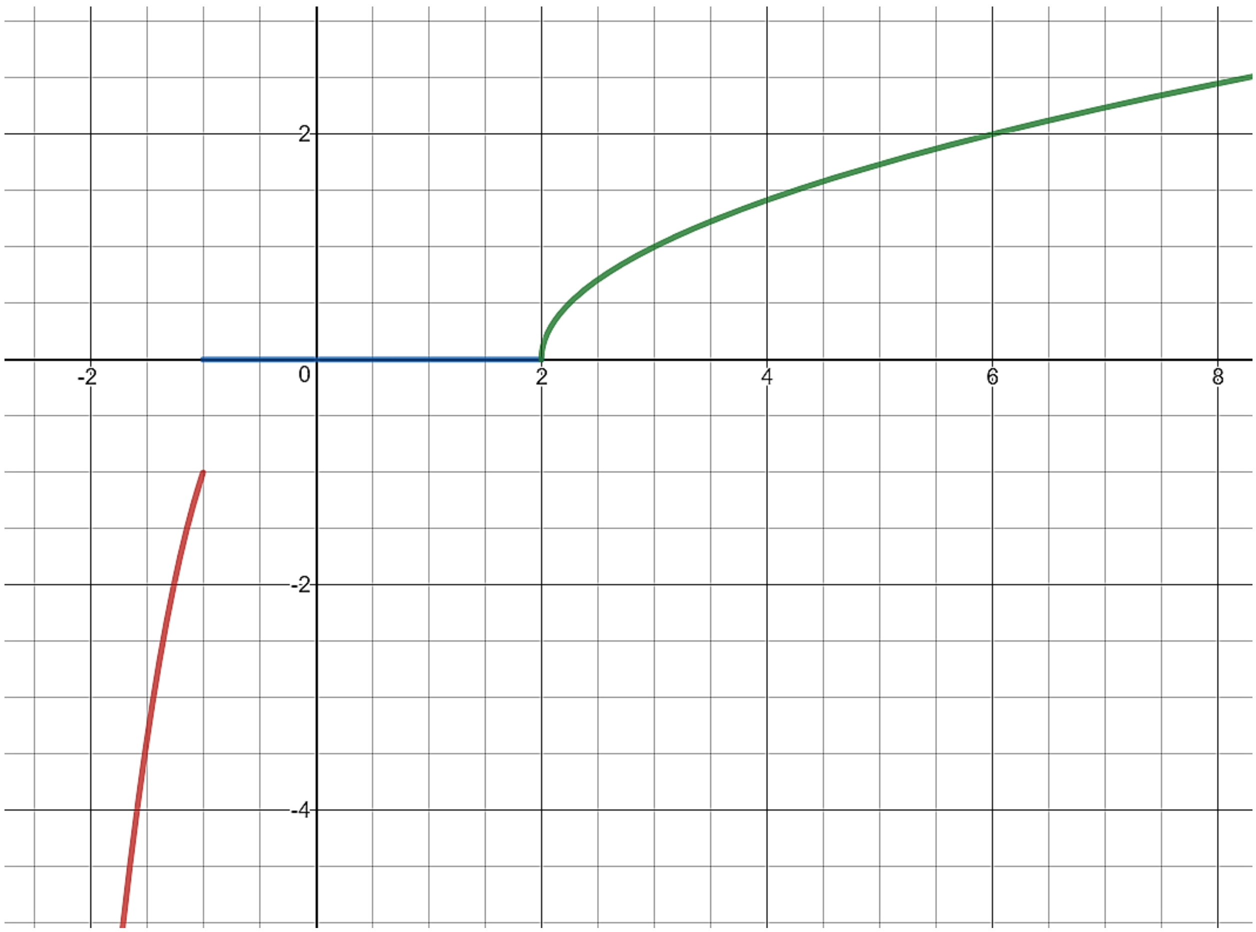
e. **False**, the notation $f''(a)$ is the second derivative of the function f

8.

$$f(x) = \begin{cases} x^3, & \text{if } x < -1 \\ 0, & \text{if } -1 \le x < 2 \\ \sqrt{x-2}, & \text{if } x \ge 2 \end{cases}$$

Also, Identify all of the discontinuities of f. Are they removable, jump, or infinite?

$x = -1$, Jump Discontinuity



9. Using the definition of a derivative, find the derivative function of:

$$f(x) = 2x^2 + 1$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= 4x$$

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10. Using the definition of a derivative, find the derivative function of:

$$g(x) = \frac{3 - x}{x}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= -\frac{3}{x^2}$$

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11. Find an equation of the tangent line to the curve $f(x) = 2x^2 + 1$ at (-2, 9). Do not use any shortcuts for finding the slope.

$$m_{tan} = 4x$$

$$f'(-2) = 4(-2) = -8$$

Point-slope Equation for the Tangent Line:

$$y - 9 = -8\big(x - (-2)\big)$$

Or, Slope-intercept form (not required)

$$y = -8x - 7$$

12. Find an equation of the tangent line to the curve $g(x) = \frac{3-x}{x}$, at (-1, -4). Do not use any shortcuts for finding the slope.

$$m_{tan} = -\frac{3}{x^2}$$

$$g'(-1) = -\frac{3}{(-1)^2} = -3$$

Point-slope Equation for the Tangent Line:

$$y - (-4) = -3(x - (-1))$$

$$y + 4 = -3(x + 1)$$

Or, Slope-intercept form (not required)

$$y = -3x - 7$$

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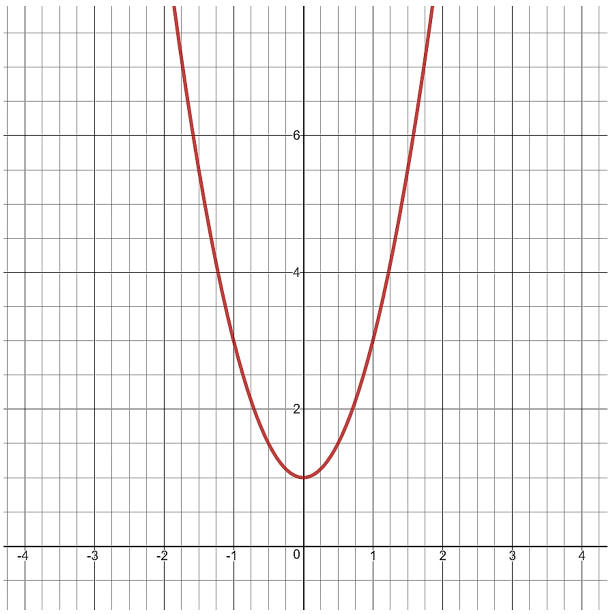
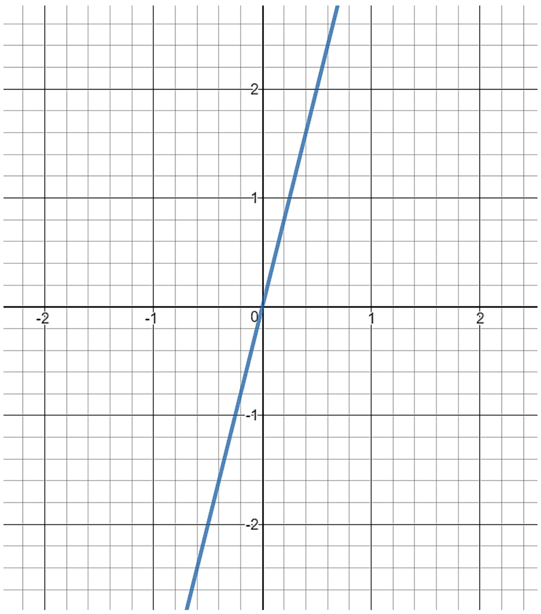
13. Sketch a graph of $f(x) = 2x^2 + 1$. Then, sketch the graph of $f'(x)$.

$$f(x) = 2x^2 + 1$$

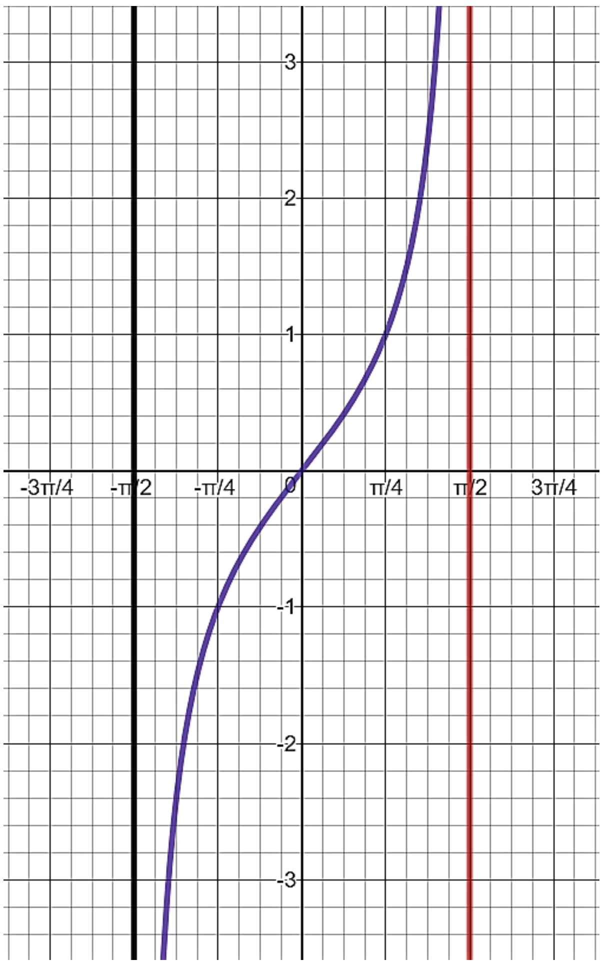


$$f'(x) = 4x$$

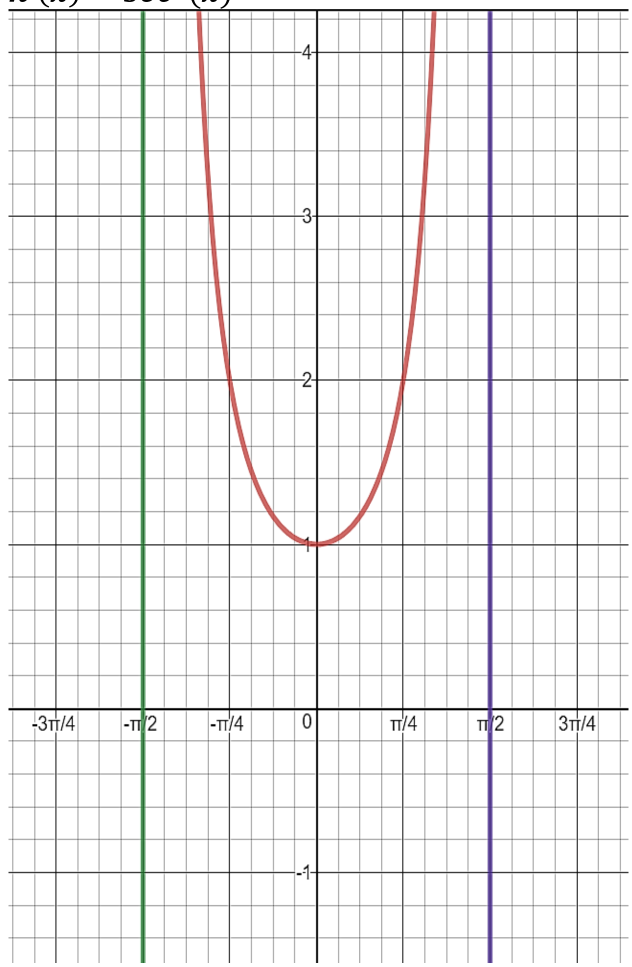


14. Sketch a graph of $h(x) = tan(x)$. Then, sketch the graph of $h'(x)$.

$h(x) = tan(x)$



$h'(x) = sec^2(x)$



15. Below is the graph of a function $g(x)$. Sketch a graph of $g'(x)$.