# NON-CARTESIAN COORDINATE SYSTEMS

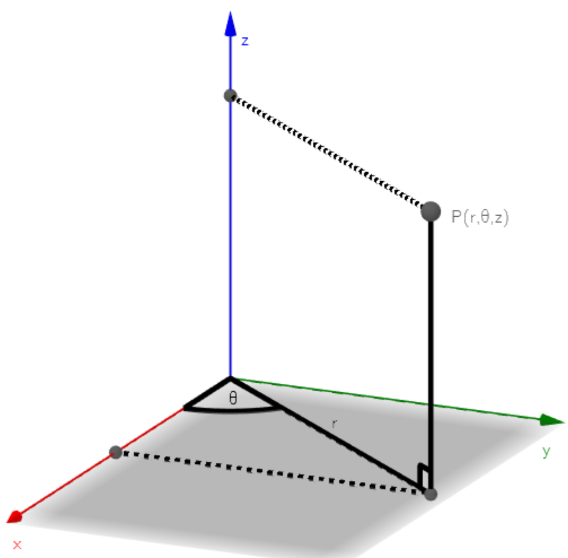
MTH 253 LECTURE NOTES

**Exploration:** In $\mathbb{R}^2$, we typically use either Cartesian or Polar coordinates. So far, we have used an analog of Cartesian coordinates. Now, we will expand on Polar coordinates with two different systems.

**Cylindrical Coordinates**

We identify every points by the ordered triple $(r, \theta, z)$, where $r$ and $\theta$ are the polar coordinates of the projection of $P$ onto the $xy$-plane, and $z$ is the directed distance from the $xy$-plane. That is, to navigate to $P$, we begin at the origin, rotate to point in the direction of $P$, move directly underneath or above $P$, and then raise or lower to the point $P$.

**Exploration:** https://www.geogebra.org/classic/X3j28ZkC




**Cylindrical → Rectangular**

**Rectangular → Cylindrical**

**Example 1.** Draw a set of coordinate axes for $\mathbb{R}^3$. Convert the rectangular point $P(-\sqrt{2}, -\sqrt{2}, 1)$ to cylindrical coordinates, and plot $P$ on your axes.

**Example 2.** Draw a set of coordinate axes for $\mathbb{R}^3$. Convert the cylindrical point $Q\left(4, \frac{5\pi}{6}, -3\right)$ to rectangular coordinates, and plot $Q$ on your axes.

**Exercise 1.** Draw a set of coordinate axes for $\mathbb{R}^3$.

a. Convert the rectangular point $P(\sqrt{3}, -1, 2)$ to cylindrical coordinates, and plot $P$ on your axes.

b. Convert the cylindrical point $Q\left(4, \frac{2\pi}{3}, -1\right)$ to rectangular coordinates, and plot $Q$ on your axes.

**Example 3.** Describe the surface whose equation is given.

a. $z = r$

b. $\theta = \dfrac{\pi}{6}$

**Exercise 2.** Describe the surface whose equation is given.

a. $r = 2$

b. $\theta = 1$

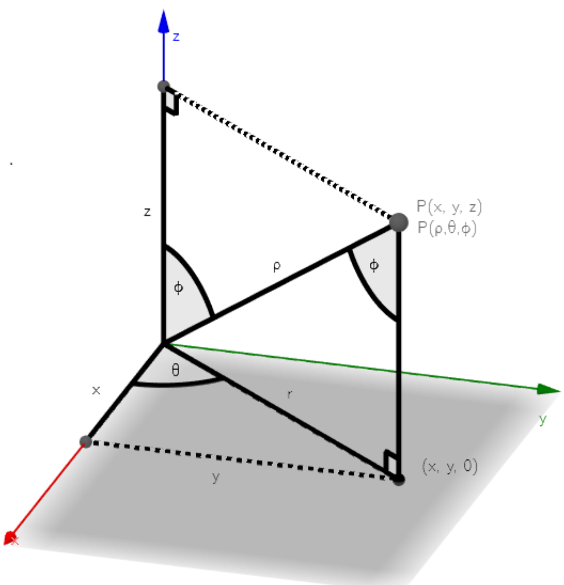
**Example 4.** Find an equation for the quadric surface $4x^2 + 4y^2 - z = 0$ in cylindrical coordinates. What kind of a quadric surface is this?

**Exercise 3.** Classify the quadric surface $3r^2 - 4z^2 = 0$ in cylindrical coordinates.

**Spherical Coordinates**

We identify every point by the ordered triple $(\rho, \theta, \phi)$, where we first rotate along the $xy$-plane a directed angle of $\theta$, then rotate downward from the $z$-axis an angle of $\phi$, then travel a distance of $\rho$ to the point $P$. Note that $\rho \geq 0$ and $0 \leq \phi \leq \pi$.

**Exploration:** https://www.geogebra.org/classic/P2RQ7SRz




**Spherical → Rectangular**

**Rectangular → Spherical**

**Example 5.** Draw a set of coordinate axes for $\mathbb{R}^3$. Convert the spherical point $P\left(4, \frac{\pi}{3}, \frac{\pi}{4}\right)$ to rectangular coordinates, and plot $P$ on your axes.

**Example 6.** Draw a set of coordinate axes for $\mathbb{R}^3$. Convert the rectangular point $Q(0, 2\sqrt{3}, -2)$ to spherical coordinates, and plot $Q$ on your axes.

**Exercise 4.** Draw a set of coordinate axes for $\mathbb{R}^3$.

a. Convert the spherical point $P\left(1, \frac{\pi}{4}, \frac{3\pi}{4}\right)$ to rectangular coordinates, and plot $P$ on your axes.
b. Convert the rectangular point $Q(-1, 1, -\sqrt{2})$ to spherical coordinates, and plot $Q$ on your axes.

**Example 7.** Given the spherical equation $\rho = \sin\theta \sin\phi$, find a rectangular equation for the surface and identify the surface.

**Exercise 5.** Given the spherical equation $\rho^2 \sin^2\phi - \rho^2 \cos^2\phi = 1$, find a rectangular equation for the surface and identify the surface.