LESSON 6

Functions of Several Variables

Contents

We've already looked at vector-functions in Two-variables. We now look at real-valued functions in Two-variables. That is, instead of the function outputting a vector, it outputs a real number.

Definition 6.0.1

A real-valued function, f, in Two-variables is a function whose domain is a subset of $\mathbb{R}^2$ (that is, a subset of $\{(x, y)|x \in \mathbb{R} \text{ and } y \in \mathbb{R}\}$) and whose codomain is $\mathbb{R}$ (whose range is a subset of $\mathbb{R}$ or whose outputs are real numbers).

In essence, we can think of real-valued functions in Two-variables as outputting the elevation of a landscape at some coordinate location in x and y. It must be a landscape since we are dealing with a function - that is, we cannot obtain two z-values for a given point in the domain of the function.

6.1 Domains and Graphing using Geogebra

Graphing a real-valued function in Two-variables in Geogebra is very simple. Open Geogebra 3D Graphing and simply type in the function and hit enter.

Example 6.1.1 Determine the domain of the function $f(x, y) = \sqrt{x^2 + y^2 - 16}$ and then use Geogebra to graph the function.

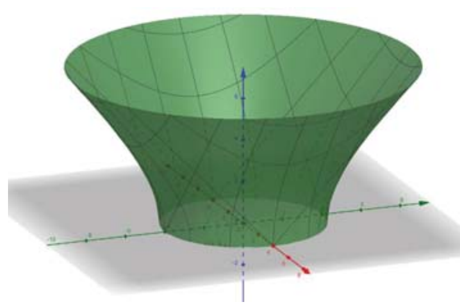


Figure 6.1.1: $f(x, y) = \sqrt{x^2 + y^2 - 16}$
View Graph Using Geogebra
https://www.geogebra.org/3d/ugkxctz9

Exercise 6.1.1 Determine the domain of the function $g(x, y) = \ln(xy)$ and then use Geogebra to graph the function.

View Graph Using Geogebra: https://www.geogebra.org/3d/tjnvvc6n

6.2 Level Curves and Contour Maps

Definition 6.2.1

The **Level Curves** of a real-valued function f in Two-variables are the 2-dimensional curves in the xy plane with equation $f(x, y) = k$ where k is a constant real number in the range of f.

Since k is an output of f in the above definition, a level curve plots all coordinate locations that have a given z-value elevation of the landscape.

Definition 6.2.2

A **Contour Map** of a real-valued function f in Two-variables is a graph of several level curves of the function.

Example 6.2.1 A contour map for a function f is shown in Figure 6.2.1 below. Use it to estimate the values of $f(1, 2)$ and $f(2, -5)$.

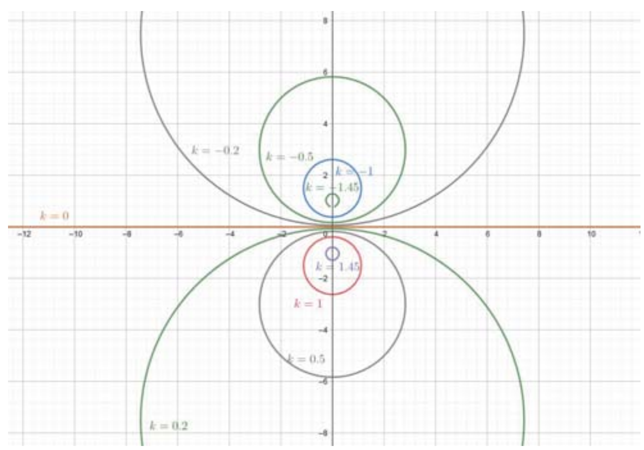


Figure 6.2.1: The Contour Map for the function f.

Graphing a contour map of a real-valued function in Two-variables in Geogebra takes a few more steps. Open Geogebra Graphing Calculator and then graph individual level curves by setting the function's expression equal to multiple k-values. You may also set k as a slider and then animate the level curves. The details will be explained in the following example.

Example 6.2.2 Use Geogebra to graph the function $f(x, y) = xy^3 - yx^3$ alongside a contour map.

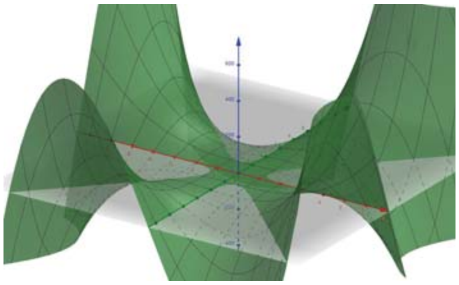


Figure 6.2.2: $z = f(x, y)$
View Graph Using Geogebra
https://www.geogebra.org/3d/kj2muxpu

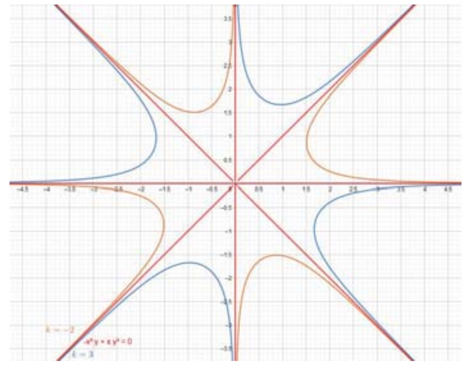
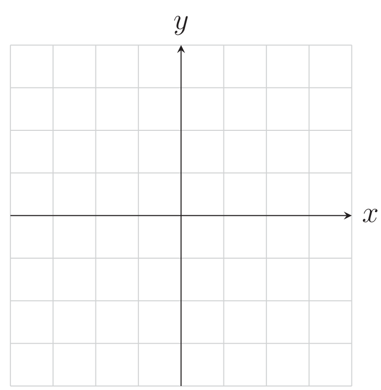
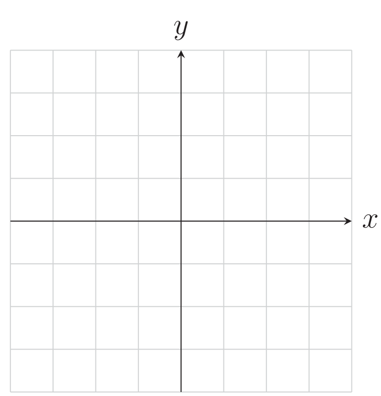


Figure 6.2.3: Contour Map of $f(x, y) = k$
View Graph Using Geogebra
https://www.geogebra.org/graphing/becmkmh6

Example 6.2.3 Draw a contour map of the function $f(x, y) = x^3 - y$ showing several level curves.



Exercise 6.2.1 Draw a contour map of the function $f(x, y) = \dfrac{y}{x^2 + y^2}$ showing several level curves.



6.3 Graphing a Function in Two-Variables by Hand

Example 6.3.1 Use the given contour maps below to sketch the graph of the associated functions.

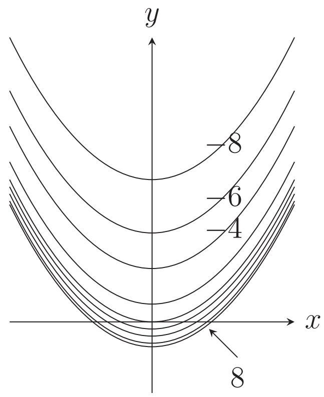
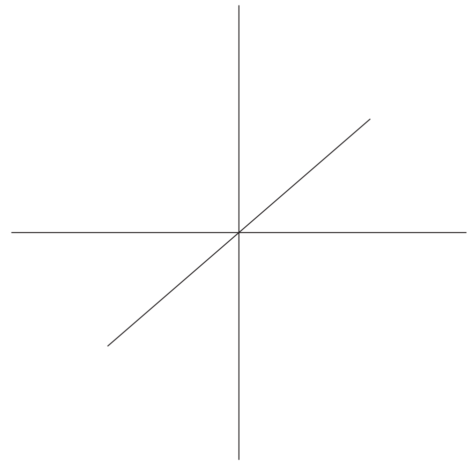


Figure 6.3.1: Contour Map (A)



Exercise 6.3.1

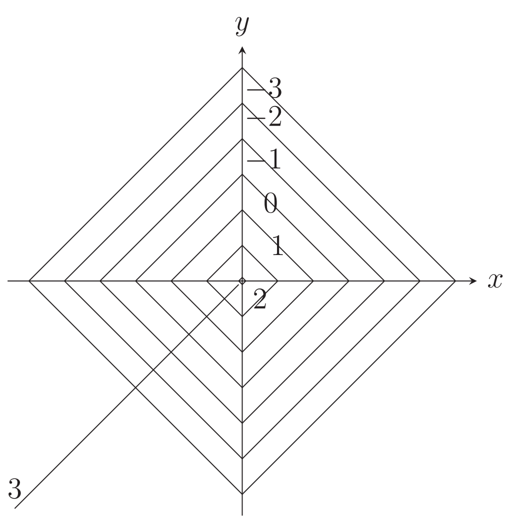
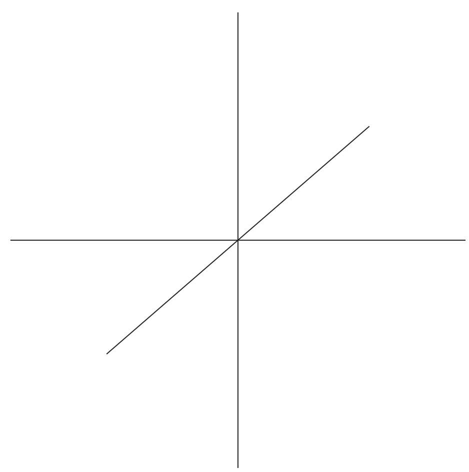


Figure 6.3.2: Contour Map (B)



6.4 Functions in Three-Variables

Definition 6.4.1

A real-valued **Function, f, in Three-Variables** is a function whose domain is a subset of $\mathbb{R}^3$ and whose range is a subset of $\mathbb{R}$.

Since the totality of the domain and range is Four-Dimensional, we cannot view a graph of f in its entirety. However, we may see its **Level Surfaces** in Three-Dimensions. Analogous to level curves in Two-Dimensions, a level surface will show us all points in Three-Space that output a given k-value in the range of the function.

Example 6.4.1 Let $g(x, y, z) = \ln(25 - x^2 - y^2 - z^2)$. Evaluate $g(2, -2, 4)$, and determine the domain and range of the function.

Example 6.4.2 Describe and graph some level surfaces to the function $f(x, y, z) = x^2 + 3y^2 + 5z^2$.

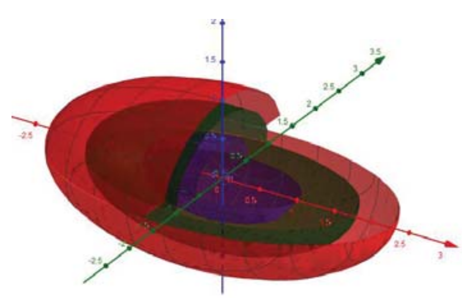


Figure 6.4.1: Level Surfaces for $f(x, y, z) = x^2 + 3y^2 + 5z^2$
View in Geogebra: https://www.geogebra.org/3d/kyq5uezy

Exercise 6.4.1 Describe and graph some level surfaces to the function $f(x, y, z) = x^2 - y^2 - z^2$.

View Graph Using Geogebra: https://www.geogebra.org/3d/e5vh9uxf