NEWTON'S LAW OF COOLING

MTH 253 LECTURE NOTES

Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. Note that this applies to warming, as well.

If we let $T(t)$ be the temperature of an object at time t and let T_s be the constant temperature of the object's surroundings, then we can write Newton's Law of Cooling as the differential equation

$$\frac{dT}{dt} = k(T - T_s)$$

Example 1. Solve the differential equation $\frac{dT}{dt} = k(T - T_s)$ using the separation of variables technique, then solve the initial value problem if we let $T(0) = T_0$.

Newton's Law of Cooling

The solution of the Newton's Law of Cooling model and initial value problem

$$\frac{dT}{dt} = k(T - T_S) \quad \text{with } T(0) = T_0$$

is $T(t) = (T_0 - T_S)e^{kt} + T_S$, where T_S is the constant temperature of the object's surroundings and T_0 is the object's initial temperature.

Example 2. A room-temperature beverage that is 72°F is placed into a refrigerator set at 35°F. After 30 minutes, the temperature of the beverage is found to be 60°F.

a. Use Newton's Law of Cooling to represent the temperature of the beverage T, measured in degrees Fahrenheit, at any moment in time t, measured in minutes.

b. What is the temperature of the beverage after 1 hour?

c. When will the beverage be ready to enjoy at a delightful 40°F?

d. Find $\lim\limits_{t\to\infty} T(t)$, and interpret in the context of the prompt.

Sous vide is a method of cooking food by vacuum-sealing the food and immersing it in warm or hot water that is maintained at a fixed temperature. This method is especially great for meat and seafood since, as long as the water is set at or just below the preferred temperature of the cooked food, the food will never overcook. After being fully cooking by sous vide, the food is then often seared to finish.

Exercise 1. A fresh-caught Coho salmon filet is vacuum-sealed and placed into a water bath set to 131°F. The initial temperature of the salmon is 35°F. According to a recipe, it takes 1 hour for the salmon to cook to the desired temperature of 125°F.

a. Find the salmon's temperature T, measured in degrees Fahrenheit, as a function of time t, measured in minutes.

b. If you aren't ready to eat yet, and you decide to leave the salmon in the water bath for 90 minutes, what will the temperature of the salmon be?

c. Find $\lim_{t\to\infty} T(t)$, and interpret in the context of the prompt.