Math 252
Final Review Key

Damien Adams

1 Conceptual Questions

1. What makes an integral an improper integral?
2. What kind of shape is used for each area approximation: Left- and right-endpoint, Trapezoidal Rule, Midpoint Rule, and Simpson's Rule?
3. When finding the volume of a solid of revolution, describe when a disk method is useful. Describe when a washer method is useful. Describe when a shell method is useful.
4. What is the relationship between S_n, M_n, and T_n?
5. When can we use L'Hôpital's Rule?
6. List the indeterminate forms.
7. Why can't we use the Fundamental Theorem of Calculus Part II to integrate $\displaystyle\int_1^3 \frac{1}{x-2}\,dx$?

2 Computational Questions

1. A rectangle has its base on the x-axis and its upper two vertices on the semicircle $y = \sqrt{64 - x^2}$. What is the largest area the rectangle can have. Use calculus and show all work in order to receive credit.
2. An object moves along a line so that its velocity at time t is $v(t) = 3t^2 - 22t + 24$ meters per second. Find the displacement and total distance traveled by the object for $0 \leq t \leq 8$.
3. Evaluate $\displaystyle\int_0^7 (x^4 - 8x + 7)\,dx$.
4. Evaluate $\displaystyle\int_0^1 (1-r)^9\,dr$.
5. Evaluate $\displaystyle\int \frac{9x^2}{\sqrt[3]{x^3+2}}\,dx$.
6. Evaluate $\displaystyle\int \sin^3 x \cos x\,dx$.
7. Evaluate $\displaystyle\int_{-1}^1 \cos x \tan x\,dx$.

8. Evaluate $\displaystyle\int \frac{6x}{\sqrt{3x^2-1}}\,dx$.

9. Evaluate $\displaystyle\int \frac{\ln(\ln x)}{x}\,dx$.

10. Evaluate $\displaystyle\int x^2 e^{-x}\,dx$.

11. Evaluate $\displaystyle\int \frac{4x+1}{x(x^2-4)}\,dx$.

    Note: This question contains a denominator that is more complicated than one you will be assessed on. However, it is a doable problem, so I leave it on here to try.

12. Evaluate $\displaystyle\int \cos^3 x\,dx$.

13. Evaluate $\displaystyle\int_4^6 \frac{2}{5-x}\,dx$. If the integral diverges, show the work that leads to your conclusion.

14. Find the area of the region bounded by the curves $y = \dfrac{1}{1+x^2}$, $y = 1+\ln(x+1)$, $x = 0$, and $x = 1$.

15. Find the volume of the solid obtained by rotating the region bounded by $y = 2x$ and $y = x^2$ about the y-axis.

16. Find the volume of the solid obtained by rotating the region bounded by $y = xe^{-x}$, $0 \le x \le 2$, about the y-axis.

17. Find an integral that represents the length of the curve $y = \ln|\sec x|$, $0 \le x \le \frac{\pi}{4}$.

18. Damien used to be a runner (not a very good one, mind you). His friend took speed readings in m/s for the second and recorded the results as follows

t	0	1	2	3	4	5	6	7	8	9	10
$v(t)$	0	8.2	12.6	14.0	16.1	18.5	18.9	20.2	20.1	16.1	3.2

    Use Simpson's Rule to estimate how far Damien sprinted before his legs gave out.