MTH 252 Lab
Numerical Approximations

Damien Adams

Purpose

We have finished learning integration strategies, but we still don't know how to integrate functions like e^{-x^2}, $\ln(\ln x)$, or $\arctan(x^2)$. Though we will not try to find antiderivatives of these, we can still approximate definite integrals.

(a) We have L_n, R_n, M_n, T_n, and S_n. Which one is generally the most accurate? Which two are the least accurate?

(b) Which is generally more accurate: T_n or M_n?

(c) L_n, R_n, M_n each use rectangles to approximate an integral. T_n uses trapezoids. What shape does S_n use to approximate a definite integral?

Prompts

1. The integral $\displaystyle\int_2^3 \frac{2}{x\ln x}\,dx$ can be found exactly. It turns out, $\star = \displaystyle\int_2^3 \frac{2}{x\ln x}\,dx$ has an exact value of $2\ln\left(\frac{\ln 3}{\ln 2}\right)$.

   a. Approximate $\star$ by rounding $2\ln\left(\frac{\ln 3}{\ln 2}\right)$ to the nearest ten-thousandth.
   b. Approximate $\star$ by computing L_4. Round your conclusion to the nearest ten-thousandth.
   c. Approximate $\star$ by computing R_4. Round your conclusion to the nearest ten-thousandth.
   d. Approximate $\star$ by computing M_4. Round your conclusion to the nearest ten-thousandth.
   e. Approximate $\star$ by computing T_4. Round your conclusion to the nearest ten-thousandth.
   f. Approximate $\star$ by computing S_4. Round your conclusion to the nearest ten-thousandth.
   g. Compare the results of the previous computations with the conclusion you found in (a). Which strategy was most accurate? Which was least accurate?

2. An antiderivative for $f(x) = e^{-x^2}$ is difficult to find, but the area underneath the curve from 0 to 1 can still be represented by $\int_0^1 e^{-x^2}\, dx$. Approximate this value to the nearest thousandth by using

   a. M_4
   b. T_4
   c. S_4

   Then find an error bound on each of the strategies used above by using a value of $K = 2$.

   d. E_M
   e. E_T

3. The velocity of Supergirl flying through the air (in km/s) is recorded every 5 seconds from the moment she takes flight. The results are provided in the table below:

t (sec)	0	5	10	15	20	25	30	35	40	45	50
$v(t)$ (km/s)	0	80	100	128	144	160	152	136	128	120	136

   Estimate the distance that Supergirl traveled (to the nearest km) by using each of the approximation strategies below.

   a. T_{10}
   b. S_{10}