MTH 252 Lab
The Fundamental Theorem of Calculus

Damien Adams

Purpose

The Fundamental Theorem of Calculus is the most important theorem of this course. Similarly, the Net Change Theorem holds the key as to how to apply integral calculus to reality.

(a) The FTC1 tells us how to differentiate an accumulation function.

(b) The FTC2 tells us how to integrate a continuous function on a closed interval.

(c) What needs to be true about $f(x)$ in order to use FTC2 on $\int_a^b f(x)\ dx$?

Prompts

1. Let $f(x) = \cos x + \sin x$. Let $g(x) = \int_0^x f(t)\ dt$.

   (a) Graph f in Desmos. Copy this graph onto your paper three times.

   (b) Evaluate $g(0)$.

   (c) Evaluate $g\left(\frac{\pi}{6}\right)$. On your first graph, draw a representation of $g\left(\frac{\pi}{6}\right)$.

   (d) Evaluate $g\left(\frac{\pi}{4}\right)$. On your second graph, draw a representation of $g\left(\frac{\pi}{4}\right)$.

   (e) Evaluate $g\left(\frac{\pi}{3}\right)$. On your third graph, draw a representation of $g\left(\frac{\pi}{3}\right)$.

   (f) Find $\dfrac{d}{dx}\left(\int_0^x f(t)\ dt\right)$.

2. Evaluate the following derivatives.

(a) $\dfrac{d}{dx}\left(\displaystyle\int_1^x \tan t \; dt\right)$.

(b) $\dfrac{d}{dx}\left(\displaystyle\int_\pi^{x^2} \tan t \; dt\right)$.

(c) $\dfrac{d}{dx}\left(\displaystyle\int_0^{\tan x} e^{t^2} \; dt\right)$.

(d) $\dfrac{d}{dx}\left(\displaystyle\int_{10}^{\cos x} \dfrac{-1}{t} \; dt\right)$.

3. Evaluate the following definite integrals.

(a) Evaluate $\displaystyle\int_1^2 (x + 2x^2 + 3x^3 + 4x^4) \; dx$

(b) Evaluate $\displaystyle\int_{-1}^1 x^2 + \dfrac{1}{1 + x^2} \; dx$

(c) Evaluate $\displaystyle\int_3^3 \tan t \; dt$

4. A soccer player runs on a field for 6 seconds with velocity (in feet per second) given by $v(t) = 2t^3 - 18t^2 + 36t$.

(a) Find the total displacement of the player in these 6 seconds.

(b) Find the total distance traveled by the player in these 6 seconds.