13.9 The Fundamental Theorems of Calculus

We state here, without the proper assumptions, the fundamental theorems to calculus. What is most important to note is that each of these fundamental theorems gives us a tool to integrate a "derivative" over a region on the left by replacing it with a quantity that involves only the values of the original function on the *boundary* of the region on the right.

The Fundamental Theorem of Calculus

$$\int_a^b F'(x)\ dx = F(b) - F(a)$$

The Fundamental Theorem for Line Integrals

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

Green's Theorem

$$\iint_D (Q_x - P_y)\ dA = \int_C P\ dx + Q\ dy$$

$$\iint_D (\operatorname{curl} \mathbf{F}) \cdot \mathbf{k}\ dA = \int_C \mathbf{F} \cdot d\mathbf{r}$$

Stokes' Theorem

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}$$

The Divergence Theorem

$$\iiint_E \operatorname{div} \mathbf{F}\ dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$$