MTH 255
Final Exam Review

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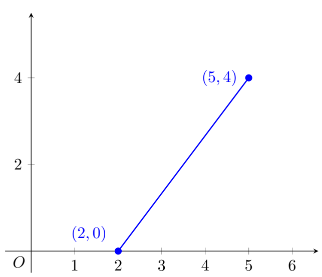
1. Sketch the vector field $\mathbf{F}(x,y) = \dfrac{y\mathbf{i} - x\mathbf{j}}{\sqrt{x^2+y^2}}$. Draw enough vectors to give a good idea for how the vector field behaves.

2. Be prepared to match vector fields provided a selection of graphs and formulas.

3. Find the gradient vector field of $f(x,y,z) = 2xe^{3z} - \frac{x}{y+4} + z\cos(2y)$.

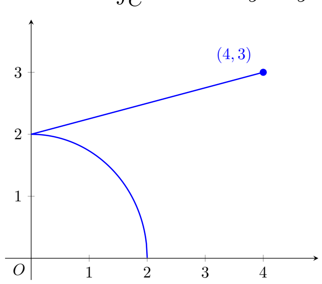
4. Find the gradient vector field of $f(x,y) = \frac{x}{y}$. Sketch 10 vectors in the gradient field to give a small idea for how the gradient vector field behaves.

5. Let C be the curve depicted below. Evaluate $\int_C xe^y \, ds$.



6. Evaluate $\int_C yz\cos x \, ds$ where C is given by $x = t, y = 3\cos t, z = 3\sin t$ and $0 \le t \le \pi$.

7. Let C be the curve depicted below. Note that C is formed by connecting a circular arc to a line segment. Evaluate $\int_C x^2 \, dx + y^2 \, dy$.



8. Let C be the closed curve formed by the arc of $y = x^2$ from the origin to $(1,1)$, the line segments connecting $(0,0)$ to $(0,1)$, and $(0,1)$ to $(1,1)$. Evaluate $\oint_C x^2y^2 \, dx + xy \, dy$ without the aid of Green's Theorem.

9. Determine whether $\mathbf{F} = ye^x\mathbf{i} + (e^x + e^y)\mathbf{j}$ is a conservative vector field or not. If it is, find a potential function for $\mathbf{F}$.

10. Let C be any path from $(-e, \pi)$ to $(2, \sqrt{2})$. Determine whether or not the following line integral is path-independent.

$$\int_C (4\cos y - 10xy - \sec^2 y)\ dx + (-4x \sin y - 5x^2 + e^y)\ dy$$

11. Let C be the path along the helix $\mathbf{r}(t) = \langle 2\sin t, 2\cos t, \frac{2}{\pi}t - 1\rangle$ from $(-2, 0, -2)$ to $(0, -2, 1)$. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \nabla\left(2xe^{3z} - \frac{x}{y+4} + z\cos(2y)\right)$.

12. Let C be the triangle with vertices $(0, 0)$, $(2, 1)$, and $(0, 1)$. Evaluate $\oint_C (x^2 + y^2)\ dx + (x^2 - y^2)\ dy$.

13. Let C be the ellipse $x^2 + 2y^2 = 2$. Evaluate $\oint_C y^4\ dx + 2xy^3\ dy$.

14. Let C be the closed curve formed by the arc of $y = \cos x$ from $\left(\frac{-\pi}{2}, 0\right)$ to $\left(\frac{\pi}{2}, 0\right)$ and the line segment connecting the endpoints of the arc. Let $\mathbf{F}(x, y) = \langle e^{-x} + y^2, e^{-y} + x^2\rangle$. Draw C, and evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

15. Let C be the closed curve formed by the arc of $y = x^2$ from the origin to $(1, 1)$, the line segments connecting $(0, 0)$ to $(0, 1)$, and $(0, 1)$ to $(1, 1)$. Evaluate $\oint_C x^2y^2\ dx + xy\ dy$ using Green's Theorem.

16. Let $\mathbf{F} = e^z\mathbf{i} + \mathbf{j} + xe^z\mathbf{k}$. Use curl to determine if $\mathbf{F}$ is conservative or not. If it is conservative, find a potential function f for $\mathbf{F}$.

17. Let $\mathbf{F}(x, y, z) = \langle e^x, e^{xy}, e^{xyz}\rangle$. Find both the curl and divergence of $\mathbf{F}$.

18. Let S be the hemisphere $x^2 + y^2 + z^2 = 4$ with $z \geq 0$, and let $f(x, y, z) = x^2z + y^2z$. Evaluate the surface integral $\iint_S f(x, y, z)\ dS$.

19. Let S be the triangular region with vertices $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$. Evaluate the surface integral of $f(x, y, z) = xy$ over S.

20. Let $\mathbf{F}(x, y, z) = \langle x, y, z^4\rangle$, and let S be he part of the cone $z = \sqrt{x^2 + y^2}$ underneath the plane $z = 1$ oriented downward. Find the flux of $\mathbf{F}$ across S.

21. Let $\mathbf{F} = e^{-x}\mathbf{i} + e^x\mathbf{j} + e^z\mathbf{k}$, and let C be the boundary of $2x + y + 2z = 2$ in the first octant. Evaluate the line integral of $\mathbf{F}$ over C, where C is oriented positively. Draw a picture to represent C.

22. Let $\mathbf{F}(x, y, z) = \langle e^{xy}\cos z, x^2z, xy\rangle$, and let S be the part of the unit sphere in front of the yz-plane. Evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

23. Let $\mathbf{F} = x^2y\mathbf{i} + xy^2\mathbf{j} + 2xyz\mathbf{k}$, and let S be the surface of the tetrahedron bounded by the coordinate planes and $x + 2y + z = 2$. Sketch S, and find the flux of $\mathbf{F}$ across S.

24. Suppose a fluid moves (in m/min) according to the velocity field $\mathbf{v} = \langle \ln(y - 2z), \frac{x}{y-2z}, \frac{-2x}{y-2z}\rangle$. Determine if $P(2, 5, 1)$ is a sink, source, or neither. Interpret your result in terms of the movement of this fluid.

25. There are several theorems that are fundamental to calculus. Below are five equations. Describe what conditions must be met in order to justify the equation between the two sides. Be prepared to provide the names of these theorems, as well as describe what each condition means (for example, if you say that a curve must be closed, be prepared to explain what "closed" means).

    (a) $\int_a^b F'(x)\ dx = F(b) - F(a)$
    (b) $\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$
    (c) $\iint_D (Q_x - P_y)\ dA = \int_C P\ dx + Q\ dy$
    (d) $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}$
    (e) $\iiint_E \operatorname{div} \mathbf{F}\ dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$