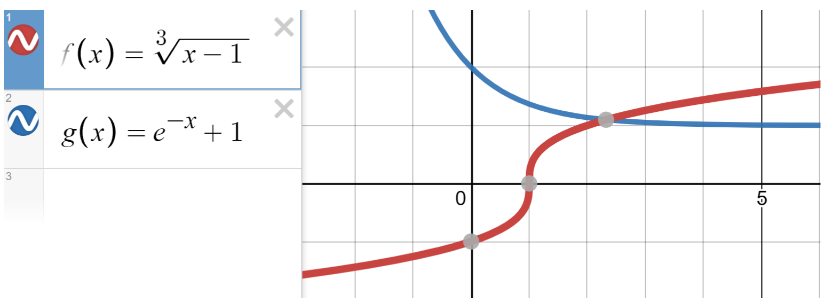
MTH 253
Midterm Review

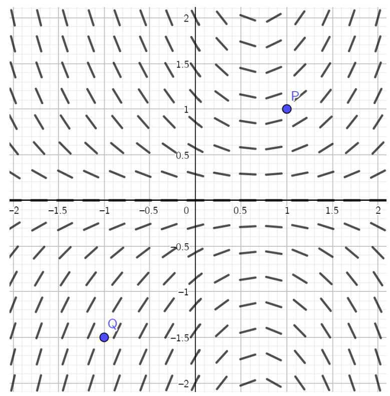
Damien Adams

1. Find the linearization to $f(x) = \sec^2 x$ at $a = \frac{\pi}{6}$.
2. Find the linear approximation to $g(t) = \ln(t^2 + 1)$ at $a = 1$.
3. Use a linear approximation to estimate $\sqrt[4]{16.05}$. Round your conclusion to the nearest thousandth.
4. Use differentials to estimate $\sqrt[4]{16.05}$. Round your conclusion to the nearest thousandth.
5. Provided are the graphs of $f(x) = \sqrt[3]{x-1}$ (in red) and $g(x) = e^{-x} + 1$ (in blue). Use x_3 in Newton's method to approximate the solution to $f(x) = g(x)$. Do all computations by hand, and show all of your work to support your conclusion. Round your conclusion to the nearest thousandth.



6. Let $f(x) = \arctan(x^2 - 2)$. Beginning with $x_1 = 1$, use x_4 in Newton's method to approximate the roots of f. Do all computations by hand, and show all of your work to support your conclusion. Round your conclusion to the nearest thousandth.
7. Verify that $y(x) = e^{-3x}$ is a solutions to the differential equation
$$y^{(4)} + y''' - 7y'' - y' + 6y = 0$$
8. Verify that $y(x) = \tan(x^3 + C)$ is a family of solutions to the differential equation
$$y' = 3x^2(y^2 + 1)$$
9. Suppose that $y(x) = \tan(x^3 + C)$ is a family of solutions to the differential equation
$$y' = 3x^2(y^2 + 1)$$
Find the particular solution such that $y(0) = 1$.
10. Sketch a slope field for the differential equation $\frac{dy}{dx} = xy - \frac{y}{x^2+1}$. Construct a set of axes with $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$, and produce a slope at each point with integer coordinates. You should have 25 slopes in total. Show all of your work that leads to the construction of your slope field.

11. Provided is a direction field for a differential equation along with the points $P(1, 1)$ and $Q(-1, -1.5)$.
    (a) Plot the solution curve that passes through P.
    (b) Plot the solution curve that passes through Q.



12. Find the general solution to the differential equation below. If necessary, express your solution implicitly. If convenient, express your solution explicitly.

$$y^3 \frac{dy}{dx} = (y^4 + 1)\cos x$$

13. Find the general solution to the differential equation below. If necessary, express your solution implicitly. If convenient, express your solution explicitly.

$$x + yy' = 0$$

14. Find the particular solution of the initial value problem

$$\frac{dy}{dx} = 3x^2(y^2 + 1) \quad , \quad y(0) = 1$$

15. Damien takes Mooncake's catfood out of the refrigerator and finds it to be 5°C. Mooncake doesn't like cold food, so Damien lets it sit out to warm in his kitchen which is 22°C. After 5 minutes, her food reaches about 7.7°C. Mooncake is picky and refuses to ear her food until it is at least 15°C.
    (a) Will Mooncake eat her food if I serve it to her in 20 minutes?
    (b) Will Mooncake eat her food if I serve it to her in 45 minutes?
    (c) What is the minimum number of minutes I need to let her food sit out before she will eat it?