MTH 252
Midterm Review

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1. Find the absolute extrema of $f(x) = \dfrac{x}{x^2 - x + 1}$ on the interval $[0, 3]$.

2. Given $f(x) = \dfrac{x}{x^2 + 1}$, find

   (a) The intervals of increase and decrease
   (b) The local extrema
   (c) The intervals of concavity
   (d) The point(s) of inflection

3. Sketch the graph of a function f satisfying all of the following properties:

   (i) $f''(x) > 0$ on $(-\infty, -1), (-1, 2)$

   (ii) $f''(x) < 0$ on $(2, \infty)$

   (iii) $f'(x) > 0$ on $(-\infty, -1), (1, 3)$

   (iv) $f'(x) < 0$ on $(-1, 1), (3, \infty)$

   (v) $f'(-1)$ does not exist

   (vi) $f(-1) = 2$

4. Find $\displaystyle\lim_{x \to 0} \dfrac{\sin 4x}{\tan 5x}$.

5. Find $\displaystyle\lim_{x \to 0^+} \sin x \ln x$.

6. Find $\displaystyle\lim_{x \to 0} \dfrac{e^{4x} - 1 - 4x}{x^2}$.

7. A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of its base is twice the width. Material for its base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

8. Find the most general antiderivative of $f(x) = 8x^9 - 3x^6 + 12x^3$.

9. Find the most general antiderivative of $f(t) = \sin t + 2\cos t$.

10. Find f if $f'(t) = 5t^4 - 3t^2 + 4$ and $f(-1) = 2$.

11. Find f if $f''(x) = 8x^3 + 5$ and $f(1) = 0$, $f'(1) = 8$.

12. A particle is moving so that $a(t) = 3\cos t - 2\sin t$ with $s(0) = 0$ and $v(0) = 4$. Find the position of the particle.

13. Write a Riemann sum for $f(x) = \sin x$ on $0 \le x \le \frac{3\pi}{2}$ with six subintervals, taking sample points to be left endpoints, then find the sum.

14. Estimate $\displaystyle\int_3^9 f(x)\, dx$ with three equal subintervals using

(a) Right endpoints

(b) Left endpoints

(c) Midpoints

where values of $f(x)$ are given in the table below.

x	3	4	5	6	7	8	9
$f(x)$	-3.4	-2.1	-0.6	0.3	0.9	1.4	1.8

15. Evaluate $\displaystyle\int 3x^2 e^{-x^3}\, dx$.

16. Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin x \sin(\cos x)\, dx$.

17. Evaluate $\displaystyle\int_{-31415926}^{31415926} \frac{x^5 \sin x \tan x |x|}{12 + x^2 + x^8}\, dx$.

18. Evaluate $\displaystyle\int \frac{\ln x}{x\sqrt{1 + (\ln x)^2}}\, dx$.

19. Evaluate $\displaystyle\int \frac{3t^2 - 2}{t^3 - 2t - 8}\, dt$.