Mth 251
Review for Exam 1

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1. The point $P(2,8)$ lies on the curve $y = x^3$.
   (a) Graph the curve $y = x^3$ and plot the point P.
   (b) If Q is the point (x, x^3), find the slope of the secant line PQ for the following values of x, rounding to *five decimal places*:

x	$m_{\text{sec}} = \dfrac{f(x) - f(2)}{x - 2}$
1.5	
1.9	
1.99	
2	cannot compute (do not write here)
2.01	
2.1	
2.5	

   (c) Taking part (b) into consideration, make an educated guess at the value of the slope of the tangent line to the curve at $P(2,8)$.

2. Repeat question 1 using $y = \dfrac{1}{x}$ and the point $P\left(2, \frac{1}{2}\right)$.

3. Sketch the graph of a function f satisfying *all* of the given conditions.

   (a) $\lim_{x \to -\infty} f(x) = -\infty$
   (b) $\lim_{x \to \infty} f(x) = \infty$
   (c) $\lim_{x \to 0^-} f(x) = -\infty$
   (d) $\lim_{x \to 0^+} f(x) = \infty$
   (e) $\lim_{x \to -2} f(x) = 1$
   (f) $\lim_{x \to 2} f(x) = -1$
   (g) $f(-2) = -1$
   (h) $f(2) = 1$

4. Sketch the graph of a function f satisfying *all* of the given conditions.

(a) $\lim_{x\to-\infty} f(x) = 3$

(b) $\lim_{x\to\infty} f(x) = -4$

(c) $\lim_{x\to0^-} f(x) = -2$

(d) $\lim_{x\to0^+} f(x) = 4$

(e) $\lim_{x\to2^-} f(x) = \infty$

(f) $\lim_{x\to2^+} f(x) = -\infty$

(g) $f(0) = 0$

5. The graph of $y = f(x)$ is shown below. Use this graph to answer the following questions.

(a) Is f continuous on $(-\infty, -4)$?

(b) Is f continuous from the left at $x = -1$?

(c) Is f continuous from the right at $x = -1$?

(d) What is $\lim_{x\to-4^-} f(x)$?

(e) What is $\lim_{x\to-4^+} f(x)$?

(f) What is $\lim_{x\to-4} f(x)$?

(g) What is $f(-4)$?

(h) Is f continuous at $x = -4$?

(i) What is $\lim_{x\to-2^-} f(x)$?

(j) What is $\lim_{x\to-2^+} f(x)$?

(k) What is $\lim_{x\to-2} f(x)$?

(l) What is $f(-2)$?

(m) Is f continuous at $x = -2$?

(n) What is $\lim_{x\to-1^-} f(x)$?

(o) What is $\lim_{x\to-1^+} f(x)$?

(p) What is $\lim_{x\to-1} f(x)$?

(q) What is $f(-1)$?

(r) Is f continuous at $x = -1$?

(s) What is $\lim_{x\to3^-} f(x)$?

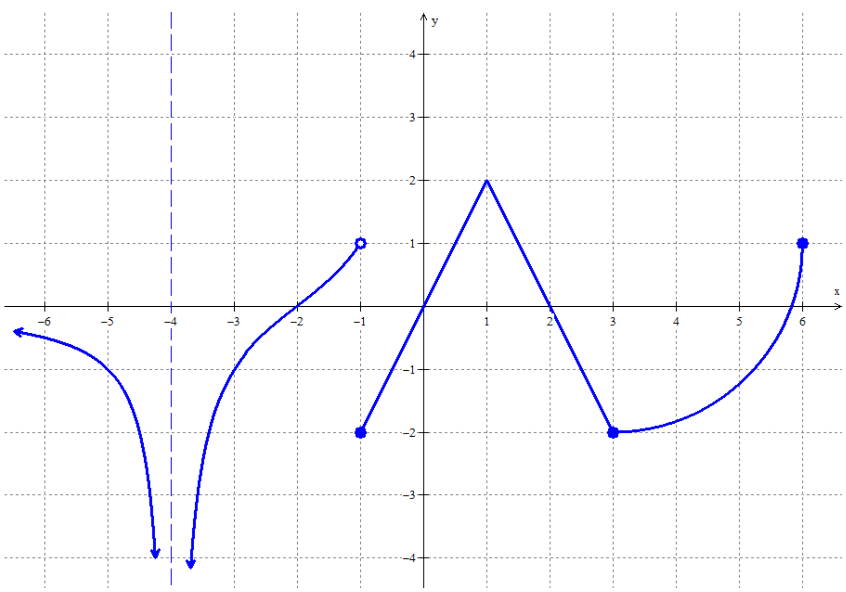
(t) What is $\lim_{x\to3^+} f(x)$?

(u) What is $\lim_{x\to3} f(x)$?

(v) What is $f(3)$?

(w) Is f continuous at $x = 3$?

(x) Where are the discontinuities of f? Identify each discontinuity as either a removable, jump, or infinite discontinuity.



6. Evaluate the limit, if it exists.

(a) $\displaystyle\lim_{x\to 3}\frac{x^2-5x+6}{2x^2-5x-3}$

(b) $\displaystyle\lim_{x\to \infty}\frac{x^2-5x+6}{2x^2-5x-3}$

(c) $\displaystyle\lim_{y\to -1}\frac{y^3+1}{y^2-1}$

(d) $\displaystyle\lim_{y\to -\infty}\frac{y^3+1}{y^2-1}$

(e) $\displaystyle\lim_{z\to 0}\frac{\cos z}{z^4-1}$

(f) $\displaystyle\lim_{t\to \infty}\frac{\sqrt{4t^2+1}}{t-1}$

(g) $\displaystyle\lim_{\alpha\to 2^-}\frac{|\alpha-2|}{\alpha^2-3\alpha+2}$

(h) $\displaystyle\lim_{\alpha\to 2^+}\frac{|\alpha-2|}{\alpha^2-3\alpha+2}$

(i) $\displaystyle\lim_{\alpha\to 2}\frac{|\alpha-2|}{\alpha^2-3\alpha+2}$

7. Below are five statements. Determine if the statement is True or False. If the statement is True, you need only write "True" and do not need to provide a justification (though one may provide partial credit). If the statement is False, write "False" and justify your answer as specifically as possible. (Do not write "T" or "F"; please write the full word)

(a) The derivative of the velocity function represents acceleration.

(b) The function $f(t)=\sqrt{4-t^2}$ is continuous from the left at $t=-2$.

(c) If a function is defined at a, it is continuous at a.

(d) If a function is continuous at a, it is defined at a.

(e) The derivative of a function f at a is the same as $f''(a)$.

8. Graph the piecewise function

$$f(x)=\begin{cases} x^3 & \text{if } x<-1 \\ 0 & \text{if } -1\le x<2 \\ \sqrt{x-2} & \text{if } x\ge 2 \end{cases}$$

Identify all of the discontinuities of f. Are they removable, jump, or infinite discontinuities?

9. Use the definition of derivative to find the derivative function of $f(x)=2x^2+1$

10. Use the definition of derivative to find the derivative function of $g(x)=\dfrac{3-x}{x}$

11. Find an equation of the tangent line to the curve $y=2x^2+1$ at $(-2,9)$. Do not use any *shortcuts* for finding the slope. (Though you can use them to *verify* your answer.)

12. Find an equation of the tangent line to the curve $y=\dfrac{3-x}{x}$ at $(-1,-4)$. Do not use any *shortcuts* for finding the slope. (Though you can use them to *verify* your answer.)

13. Sketch a graph of $f(x)=2x^2+1$. Then sketch a graph of $f'(x)$.

14. Sketch a graph of $h(x)=\tan x$ on the interval $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$. Then sketch a graph of $h'(x)$.

15. Below is the graph of a function $g(x)$. Sketch a graph of $g'(x)$.