

# MTH 251

## Derivatives Review

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### Questions

1. Differentiate  $y = x^2 \sin \pi x$
2. Find  $y'$  if  $y = \frac{t^4-1}{t^4+1}$ . *Please simplify.*
3. Differentiate  $y = \frac{e^x}{x^2}$ . *Please simplify.*
4. Find  $y'$  if  $y = 3^{x \ln x}$ . *Please simplify.*
5. Find  $f'(r)$  if  $f(r) = \ln(r \ln r)$ . *Please simplify.*
6. Find  $\frac{dy}{dx}$  if  $y = \log_5(1 + 2x + 3x^2)$ . *Please simplify.*
7. Differentiate  $y = \tan^2(\sin \theta)$ . *Please simplify.*
8. Find  $y''$  if  $x^6 + y^6 = 1$ . *Please simplify.*
9. Find  $\frac{dy}{dx}$  if  $\sin xy = x^2 - y$
10. Find  $\frac{dy}{dx}$  if  $y + x \cos y = x^2 y$
11. Differentiate  $y = x \sin(x^2)$ . *Please simplify.*
12. Differentiate  $f(x) = \cos x \tan(x^2 - 1)$ . *Do not simplify.*
13. Find  $f'(t)$  if  $f(t) = \arctan(\arcsin \sqrt{t})$ . *Do not simplify.*
14. Find  $g'(x)$  if  $g(x) = x \arctan(4x)$ . *Please simplify.*
15. If  $f(t) = \sqrt{4t+1}$ , find  $f''(2)$
16. If  $g(\theta) = \theta \sin \theta$ , find  $g''\left(\frac{\pi}{6}\right)$
17. Find an equation of the line tangent to the curve  $x^2 + 4xy + y^2 = 13$  at  $(2, 1)$
18. Find an equation of the line tangent to the curve  $y = (2+x)e^{-x}$  at  $(0, 2)$
19. Find an equation of the line tangent to the curve  $y = \sqrt{1+4\sin x}$  at  $(0, 1)$
20. A particle moves on a horizontal line so that its position at time  $t$  is  $s(t) = t^3 - 12t + 3$ , where  $t \geq 0$ ,  $s$  is measured in meters, and  $t$  is measured in minutes.
  - (a) What are the units of the position, velocity, and acceleration?
  - (b) Find the velocity of the particle at time  $t$
  - (c) Find the acceleration of the particle at time  $t$
  - (d) Find the velocity of the particle at 3 minutes
  - (e) When is the particle not moving?
  - (f) When is the particle moving forward?

(g) What is the position of the particle when the acceleration is 0?

21. A bacteria culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour, the population has increased to 360 cells. Suppose that the number of bacteria in the culture after  $t$  hours is given by  $n(t) = 200(3.24)^t$ .

(a) Find the number of bacteria after 4 hours

(b) What are the units of the derivative?

(c) Find the rate of growth after 4 hours

## Key

- $y' = \pi x^2 \cos \pi x + 2x \sin \pi x$
- $y' = \frac{8t^3}{(t^4+1)^2}$
- $y' = -\frac{2xe^{\frac{1}{x}} + e^{\frac{1}{x}}}{x^4}$
- $y' = \ln 3 \cdot 3^{x \ln x} (\ln x + 1)$
- $f'(r) = \frac{1}{r} + \frac{1}{r \ln r}$
- $\frac{dy}{dx} = \frac{6x+2}{\ln 5(1+2x+3x^2)}$
- $\frac{dy}{d\theta} = 2 \tan(\sin \theta) \sec^2(\sin \theta) \cos \theta$
- $y'' = -\frac{5x^4}{y^{11}}$
- $\frac{dy}{dx} = \frac{2x - y \cos xy}{x \cos xy + 1}$
- $\frac{dy}{dx} = \frac{2xy - \cos y}{1 - x \sin y - x^2}$
- $\frac{dy}{dx} = 2x^2 \cos(x^2) + \sin(x^2)$
- $f'(x) = -\sin x \tan(x^2 - 1) + 2x \cos x \sec^2(x^2 - 1)$ <sup>21.</sup>
- $f'(t) = \frac{1}{1 + (\arcsin \sqrt{t})^2} \cdot \frac{1}{\sqrt{1-t}} \cdot \frac{1}{2} t^{-\frac{1}{2}}$
- $g'(x) = \frac{4x}{1+16x^2} + \arctan(4x)$
- $-\frac{4}{27}$
- $\sqrt{3} - \frac{\pi}{12}$
- $y - 1 = -\frac{4}{5}(x - 2)$
- $y = -x + 2$
- $y = 2x + 1$
- (a) Meters, m/min, m/min<sup>2</sup>  
(b)  $v(t) = 3t^2 - 12$   
(c)  $a(t) = 6t$   
(d) 15 m/min  
(e) At 2 minutes, the particle is at rest  
(f) After 2 minutes  
(g) 3 meters in the positive direction
- (a) About 22040 bacteria  
(b) Bacteria per hour  
(c) About 25910 bacteria/hr