MTH 251
LAB §4.2 & 4.3

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1. Consider the function $f(x) = 2x^3 - 3x^2 - 36x + 1$ with domain $[-4, 0]$.
   a. Does f have a global maximum?
   b. Does f have an absolute minimum?
   c. What theorem justifies your conclusion in parts (a) and (b)?
   d. Is there a difference between an absolute extremum and a global extremum?
   e. Find the global extrema of f.

2. Consider the function $g(t) = \frac{1}{3}\sqrt{-t^2 + 4t + 77}$
   a. What is the domain of g?
   b. Graph g using Desmos.
   c. Since g is continuous on its domain, it must have a global maximum and a global minimum. In how many places does g attain its global maximum? In how many places does g attain its global minimum?
   d. Use calculus to find the global extrema of g. Where do these extrema occur? (Note that these are two separate questions)

3. It is said that Zeus gave birth to the goddess of wisdom, Athena, when she burst forth from his skull. This was not an immediate process, and it started when Zeus had a splitting (get it?) headache.
   Zeus' pain from zero to four hours can be modeled by the function below. In this model, 0 represents no pain, and 1 represents a pain so great that a god would pass out.
   $$p(t) = \frac{(5t + 1)(8 - 2t)}{(t + 2)(t + 3)^2}$$
   a. What is the domain of p (as described in the description of the model)?
   b. What are the zeros of p (in the domain of p)?
   c. Graph p using Desmos. Use this graph to answer the following questions.
   d. Find the global maximum of p.
   e. At what time does the global maximum occur?
   f. What do you think happened when this maximum was achieved?

4. Let $f(x) = \frac{x^2 - 1}{x^3}$
   a. Find $f'(x)$.
   b. What are the critical numbers of f?
   c. Create a sign chart for f'.
   d. Identify the intervals on which f' is positive.
   e. Identify the intervals on which f is increasing.
   f. Is there a correlation between the last two parts?
   g. Identify the intervals on which f' is negative.
   h. Identify the intervals on which f is decreasing.
   i. Is there a correlation between the last two parts?
   j. Identify all of the local extrema of f.

5. Find all of the local extrema of $f(x) = x\sqrt{2 + x}$.