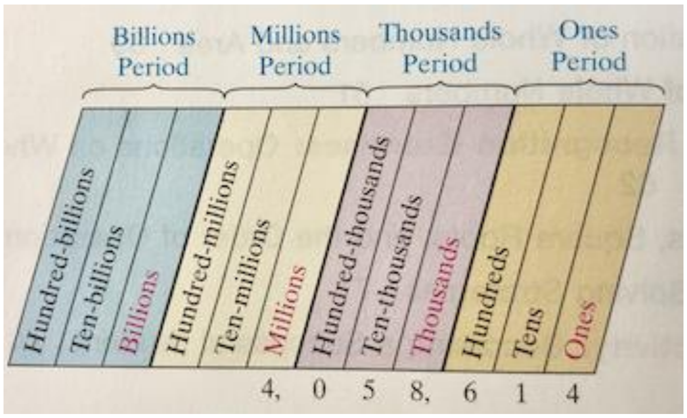
Math 20 Class Notes
Whole Numbers Basics: Section 1.1

Vocabulary

1. Digits (0,1,2,3,4,5,6,7,8,9).
2. Natural Numbers are counting numbers without zero, {1,2,3, …}.
3. Whole Numbers are counting numbers with zero, {0,1,2,3, …}.
4. Place Value (digit's value and location) & Standard Notation (using digits only)



Examples:

1. What is the place value of the digit 4 in the number 23,490?
2. What is the place value of the digit 8 in the number 58,649?

Word Names

1. Write the words using three-digit sets called periods (ones, hundreds, thousands, …)
2. Use a comma after each digit set (ones, thousands, millions, …)
3. Do NOT use the word "and" as we are saving it for decimals
4. Changing from word name to place name and vice versa.
5. Expanded Form or Notation (362 is 3 hundreds + 6 tens + 2 ones or 300 + 60 + 2).

Examples:

1. What is the word name of the number 1,364?
2. What is the word name of the number 20,692?

Inequalities

1. Discuss the meaning of the inequality symbols < and >
2. Demonstrate how to compare two whole numbers using the inequality symbols and the number line

Examples: Insert the correct inequality symbol between the two given numbers and sketch the two numbers on a number line.

1. 31 ____ 51

2. 115 ____ 92

Math 20 Class Notes
Addition & Subtraction of Whole Numbers: Sections 1.2 – 1.3

1. Addition
   a. Vocabulary
      i. **Addends** – The numbers you are adding
      ii. **Sum** – The result of addition
   b. Addition using a number line
      1. Adding to a whole number on the number line moves the value to the right.
   c. Addition using the vertical format
   d. Estimating the sum with **Front Rounding** as a check step

   Examples:
      1. Add the numbers 243 and 692, then check the sum using front rounding.
      2. Add the numbers 4671 and 3948, then check the sum using front rounding.

   e. Addition using a scientific calculator
   f. Properties of Addition.
      i. **Commutative Property of Addition** – You can add two numbers in either order and the sum is the same
      ii. **Associative Property of Addition** – When adding three numbers, you can group (thus add) the first two numbers, then add the third to the sum. Or you can add the last two numbers, then add the first number to that sum. In either case, the final sum is the same.
      iii. **Addition Property of Zero** – Adding zero to a number does not change that number
   g. Define "polygon" and "perimeter", and then use addition to find the perimeter of rectangles and squares
   h. Solving appropriate application problems (lighthouse on a cliff and making a deposit)

2. Subtraction
   a. Vocabulary
      i. **Minuend** – The first number in a subtraction problem
      ii. **Subtrahend** – The second number in a subtraction problem
      iii. **Difference** – The result of a subtraction problem
   b. Subtraction using a number line
      1. Subtracting from a whole number on the number line moves the value to the left.
   b. Vertical method of subtraction
   c. Checking the difference using addition
   d. Estimating the difference with **Front Rounding** as a check step
   e. Subtraction on a scientific calculator
   f. Calculations including both addition and subtraction (left to right)
   g. Solving appropriate application problems (how far are two cars apart based on distance from an observer and cashing a check)

   Examples:
      1. Subtract the numbers 648 and 517, then check the difference using front rounding.
      2. Subtract the numbers 3982 and 2648, then check the difference using front rounding.

Math 20 Class Notes
Rounding and Whole Numbers: Section 1.4

Rounding

We use tactics like rounding to estimate numbers and measurements. People use some form of rounding all the time. A carpenter "rounds up the length of a board" so they can look for a board just a bit longer to use from the lumber available. Saves wood. Customers at a supermarket round the prices of the items they put in their shopping cart to keep from the dreaded "not enough money for the food in my cart" embarrassment. There are many more examples.

Rounding tactics

- **Rounding Up**:
    - To round a number "up" we must decide which place value we are rounding to.
    - For example, we might round 3075 to the hundreds place.
    - Since we are rounding "up", we simply make the value in the hundreds place one digit larger and make all the digits to the right of that value zeroes.
    - In our example, we round 3075 "up" to 3100.
- **Rounding Down**:
    - To round a number "down" we must decide which place value we are rounding to.
    - For example, we might round 852 to the tens place.
    - Since we are rounding "down", we simply make all the digits to the right of that value zeroes.
    - In our example, we round 852 "down" to 850.
- **Rounding to the Nearest**:
    - In this tactic, we first need a little assistance. We take the digits 0 through 9 and break them up into two pieces: {0,1,2,3,4} and {5,6,7,8,9}
    - We determine where we wish to round the number "to the nearest", the next digit to the right is called the **test digit**.
    - Since we are rounding "up", we simply make the value in the hundreds place one digit larger and make all the digits to the right of that value zeroes.
    - For example, we might round 1493 to the nearest hundreds place. This means the "test digit" is 9. This means we round "up" to the hundreds place and get 1500.
- **Front Rounding**:
    - Front Rounding simply means we are rounding to the largest place value in a particular number. The "front" in front rounding is all about that <u>largest</u> place value in a number, no other place value.
    - For example, we would front round 852 to the hundreds place and get 900.
    - Another example, we would front round 1439 to the thousands place and get 1000.
    - And, as we have already studied, we use front rounding as a simply way to estimate values used in arithmetic, often as a "check step".

Math 20 Class Notes
Multiplication and Division of Whole Numbers: Sections 1.5 & 1.6

1. Multiplication
   a. Vocabulary
      1. **Factor**(s): The two (or more) numbers you are multiplying.
      2. **Product** – The result of multiplying two (or more) numbers.
   b. Show various symbols that are used to indicate multiplication.
   c. Multiplication with the vertical method.
   d. Estimating using **Front Rounding** as a check step.
   e. Multiplication on a scientific calculator.
   f. Properties of Multiplication.
      1. **Commutative Property of Multiplication** – You can multiply two numbers in either order and the product is the same
      2. **Associative Property of Multiplication** – When multiplying three numbers, you can group (thus multiply) the first two numbers, then multiply the third to the product. Or you can multiply the last two numbers, then multiply the first number to that product. In either case, the final product is the same.
      3. **Distributive Property** (of Multiplication over Addition) – When multiplying a number by a sum, you can "distribute the factor" by multiplying it to both addends of the sum.
      4. **Multiplication Property of Zero** – Multiplying any number by zero has a product of zero.
      5. **Multiplication Property of One** — Multiplying any number by one has a product of the original number.
   g. Define "area" and use multiplication to find the area of rectangles and squares.

2. Division
   a. Vocabulary
      1. **Dividend** - The number being divided.
      2. **Divisor** – The number dividing by: In the example $45 \div 9$, 45 is the dividend and 9 is the divisor.
      3. **Quotient** – The result of division, when the remainder is zero.
      4. **Partial Quotient** – The result of division, when the remainder is NOT zero.
   c. Long division of whole numbers and division on a scientific calculator.
   e. Estimating the quotient using **Front Rounding** as a check step. Be sure to <u>round both the dividend and the divisor either up or down</u>.
   f. **Properties of Division**
      1. Any whole number divided by one is the whole number.
      2. Any whole number divided by itself is one.
      3. Zero divided by any whole number is zero.
      4. Any whole number divided by zero is "undefined".

<u>Examples</u>:

1. Multiply the numbers 67 and 28, then check the product using front rounding.
2. Multiply the numbers 316 and 104, then check the product using front rounding.
3. Divide 6102 by 54 using long division, then check the quotient using front rounding.
4. Divide 1932 by 85 using long division, then check the quotient using front rounding.

Math 20
Long Division of Whole Numbers: Steps

Many students have difficulty learning long division.

One way you could look at long division is similar to multiplication. Note 3×5 is the same as 3 copies of 5 added (5 + 5 + 5), or "multiplication is repeated addition", we can look at long division similarly, or "long division is repeated subtraction".

Let's look at the steps using an example:

1) First, we need to change a division problem in horizontal format to long division. Consider the problem $3234 \div 22$. In this problem, we call 3234 the dividend and 22 the divisor. To change the problem to long division, we put the dividend 3234 inside the long division symbol (or "the hut", as Lewis Black calls it) and the divisor on the left side of the "hut".

   The result:

2) Now, count the number of digits in the divisor 22. On the dividend 3234, start from the left of the number and count the same number of digits inwards, which would give you 32. Now, ask yourself "How many times can I take 22 out of 32?" Of course, the answer is 1. Then, you would take $1 \times 22 = 22$ and write it under the "32" in 3234 and place a 1 above the "hut" over the 2 in 3234.

   The result:

3) Subtract the 22 from 32 as shown below.

   The result:

4) The difference is 10. Now, since we are not yet done, "bring down the next digit (3)", as shown below.

   The result:

5) Now ask "How many times can I take 22 from 103?" We can see the answer is 4. Since we know that $4 \times 22 = 88$, we subtract 88 from 103 to get 15.

   The result:

6) We can now bring down the last digit of the dividend (4) to get 154. So, we might guess that we can get 8 copies of 22 out of 154 and proceed accordingly. But we would find that $8 \times 22 = 176$, too large to subtract from 154. To fix this, we try 7, and see $7 \times 22 = 154$.

The result:

7) In this case, when we subtract the last time, the result is 0. This last difference is called the **remainder**. And we see that our **quotient** (What we call the result of division) is 147.

The result:

Some notes:

- If, after you "bring down" a digit and there not enough to take any copies of the divisor out, bring down another digit. This applies in Step 2 above, also.
- Once you bring down the last digit, whatever you get from the subtraction is your remainder.
- If there is a remainder that is not zero, report the final result as "Quotient" R "Remainder". An example might be $2750 \div 61 = 45R5$ (check it for yourself!).

Math 20 Class Notes
Exponents, Square Roots, and Order of Operation: Section 1.7

Exponents

1. Discuss the idea of **exponentiation as repetitive multiplication**.

$$\boxed{3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$$

   a. Investigate examples of exponentiation with whole number exponents
   b. Define:
      i. **Base** – This is the number being multiplied by copies of itself (the "3" above)
      ii. **Exponent** – This is the number of copies of the base that we multiply (the "5" above)
      iii. **Power** – This is the base and exponent together (the "3^5" above)
      iv. **Powers of 10** – This is when the base is 10.
      v. **Powers of 1** – This is when the base is 1.
   c. Demonstrate evaluating a power "by hand" and with a calculator.
   d. Define:
      e. **Squared** – A number is said to be "squared" if the exponent is 2. This comes from the formula for the area of a square.
      f. **Cubed** – A number is said to be "cubed" if the exponent is 3. This comes from the volume of a cube.
2. Investigate the number **1 as a base** ($1^n = 1$).
3. Investigate the number **10 as a base**.
   a. Multiplying by 10 "adds a zero" to the right end of the number.
   b. Dividing by 10 "subtracts a zero" from the right end of the number.
   c. The net effect is the movement of the decimal point, as we will study more when we study decimals.

Examples:

Determine the exponent, base, and power of 5^4

Determine the exponent, base, and power of 12^7

Write out the power 4^5 in expanded form, then evaluate it

Write out the power 2^6 in expanded form, then evaluate it

Square Roots

1. The square roots of the number is that number that is squared to get the original number.
2. The easiest way to find a square root is to use a scientific calculator (demonstrate). Some square roots are easily recognizable:

Number	**1**	**4**	**9**	**16**	**25**	**36**	**49**	**64**	**81**	**100**
Square Root	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

Order of Operations (PEMDAS)

1. Order of Operations
   a. **Parentheses and grouping symbols**: Do the work inside of a grouping symbol first. All work inside parentheses must also follow Order of Operations.
   b. Do **exponents** next.
   c. Do **multiplication and division** (left to right).
   d. Finally, do **addition and subtraction** (left to right).
2. Show how a "grouping symbol" changes the order of operation in an expression.
3. Discuss the various grouping symbols used in math, including fraction bars.
4. Demonstrate using a scientific calculator with multiple examples.
5. Demonstrate finding the average of several numbers using the skills in this chapter.

Examples:

Evaluate the given expression ONE STEP AT A TIME using Order of Operations:

$$2(3^2 + 7) - 14$$

$$\frac{6^2 - 6}{5^2 - 10}$$

Math 20 Class Notes
Problem Solving Tactics: Section 1.8

Problem Solving

- Note Table 1.5 on page 71 of the text.

Table 1-5

Operation	Key Word or Phrase
Addition	Sum, added to, increased by, more than, plus, total of
Subtraction	Difference, minus, decreased by, less, subtract
Multiplication	Product, times, multiply
Division	Quotient, divide, per, shared equally

- Use problems to demonstrate the **Problem Solving Tactics**, as shown below:
  - Read the problem "normally"
  - Re-read the problem, and write down important information and question to answer.
  - As appropriate, draw a picture, table, or graphic of the situation, labeling with the known information.
  - Look for patterns or relationships in the information from the problem to help with setting up and equation or expression.
  - Set up an expression or equation in terms of the information you have been given.
  - Complete your calculations and answer the problem's question in a sentence.
  - Check your solution to see if it "makes sense", both mathematically and realistically.

- Discuss **important versus unimportant information**.
  - When deciding if information is important to solving a problem, ask yourself "Will this information help me solve the problem, or is it just extra unneeded data?" If it helps you solve the problem, it is needed.

- Discuss **implicit versus explicit** information.
  - **Explicit information** is information you are told "up front". You don't have to ask for it or go "searching" for it. An example would be a problem where you are told the starting salary of someone who is about to get a 10% raise. The starting salary is explicit, because you are told its value.
  - **Implicit information** is information that you need and have to go look for. An example would be a formula for the area of a figure that you are not given in the problem.

Math 20 Class Notes
Fraction Basics: Section 2.1

Definitions

- **Integers** – The set of numbers consisting of whole numbers, their opposites, and (of course) zero.
- **Fraction** – A quotient of two integers, where the second integer (called the denominator) is NOT zero.
- **Numerator** – The "top" of the fraction.
- **Denominator** – The "bottom" of the fraction.
- **Proper fractions** – Fractions whose numerator is smaller in value than the denominator
- **Improper fractions** – Fractions whose numerator is larger in value than the denominator
- **Mixed numbers** – The sum of an integer and a proper fraction.
- **The result of long division** of integers is:
    - Another integer (remainder = 0)
    - A mixed number (remainder ≠ 0)
- **Using the fraction key** (cover when appropriate to the discussion)
    - To enter fractions on the calculator
    - To simplify fractions
    - To perform operations on fractions

Converting Between Types of Fractions

- We change improper fractions to mixed numbers using long division, where the remainder is "put over the divisor" to create the fraction part of the mixed number.

$$Quotient + \frac{remainder}{divisor}$$

- We change mixed numbers to improper fractions using the "word formula":

$$\frac{Whole\ number \times Denominator + Numerator}{Denominator}$$

Simplifying Fractions

- We simplify fractions by "canceling out" common factors
- We can also simplify fractions using our calculator and the fraction key

Fractions and the Number Line

- We can us a number line to "plot" fractions and mixed numbers to help us understand their order.

Math 20 Class Notes
Prime Numbers and Factorization: Section 2.2

Prime Factors

- Discuss prime and composite numbers.
  - **Prime numbers** are whole numbers whose only factors are one and itself.
  - **Composite numbers** are whole numbers that are not prime (they have other factors besides one and itself).
  - A good time for the NOVA video on the **"Twin Prime Conjecture"**.

Divisibility Tests

- Discuss **what it means for a number to be "divisible" by another** (when you divide, there is no remainder).
- Since "divisible" is about division, the second number is called the divisor.
- Remind the class about "even" and "odd" numbers.
- **Divisibility tests**
  - **Divisible by 2**: Numbers ending (one's place) in even digits are divisible by 2.
  - **Divisible by 3**: If the sum of the digits of a number is a multiple of 3, the number is divisible by 3.
  - **Divisible by 5**: If the last digit (one's place) is either 0 or 5, the number is divisible by 5.
  - **Divisible by 6**: If a number is divisible by BOTH 2 and 3, it is divisible by 6.
  - **Divisible by 9**: If the sum of the digits of a number is a multiple of 9, the number is divisible by 9.
  - **Divisible by 10**: If a number ends with a 0, it is divisible by 10.
  - **Composite Rule**: If a number is divisible by two numbers, it is divisible by the product of those two numbers. (See the rule for 6 above)
  - **A "Universal Divisibility Test"**: If one number divided by a second number has a remainder of zero, the first number is divisible by the second number.
  - **Dividing two numbers that end in zero** – we remove ending zeroes in the divisor and the same number of ending zeroes in the dividend.

Primes, Composites, and Prime Factorization

- Demonstrate prime factorization using a **factoring tree**.
- Demonstrate prime factorization using a **division ladder**.
- Discuss the **Fundamental Theorem of Arithmetic** (every composite number has exactly one unique prime factorization).
- Demonstrate how to find all the possible factors (prime or composite) of a whole number.

Math 20 Class Notes
Simplifying Fractions: Section 2.3

Equivalent Fractions

- **Equivalent Fractions** are two (or more) fractions that have the same value.
- For example, the fractions $^1/_2$, $^2/_4$, $^3/_6$, and $^4/_8$ are all equivalent fractions.

Simplifying Fractions

- The **Fundamental Principle of Fractions**: Let c be a common factor of both the numerator and denominator. Then:

$$\frac{a \times c}{b \times c} = \frac{a}{b} \times \frac{c}{c} = \frac{a}{b} \times 1 = \frac{a}{b}$$

- We **simplify fractions by removing a common factor** from both the numerator and denominator. We then repeat the process until all the common factors have been removed (cancelled).

$$\frac{36}{54} = \frac{12 \times 3}{18 \times 3} = \frac{12}{18} = \frac{2 \times 6}{3 \times 6} = \frac{2}{3}$$

- Of course, if we <u>first</u> determined ALL the common factors, we could then just "factor out" these common factors all at once, cancel, and then be done!

The Greatest Common Factor (GCF)

- The largest common factor of two numbers is called the **Greatest Common Factor**, or GCF. Using factoring trees, we can find the GCF:
  - Factor both numbers into their prime factors.
  - Identify ALL the common factors of both factorizations.
  - Calculate the GCF by multiplying the common factors.
- The GCF is that collection of factors we need when simplifying a fraction completely, as discussed above.

Math 20 Class Notes
Multiplying and Dividing Fractions: Sections 2.4 & 2.5

Multiplying and Dividing Fractions

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

- **We multiply fractions by multiplying "straight across"** (numerator times numerator and denominator times denominator).
- **We divide two fractions by changing the problems into multiplication FIRST, then multiply**
- When multiplying or dividing with mixed numbers, **always change the mixed numbers to improper fractions first**, and then proceed. If necessary, you can change the results back to a mixed number.

Order of Operations: PEMDAS

- **P**arentheses and other grouping symbols
- **E**xponents and square roots
- **M**ultiplication and **D**ivision (left to right)
- **A**ddition and **S**ubtraction (left to right)

Order of Operations – Average

- Demonstrate the use of Order of Operations with fractions:
    - Calculating an average (arithmetic mean) as the sum of all the data divided by the number of data values
    - Finding the area of a triangle from the formula **A = ½ bh**

Problem Solving Tactic

- **Read the problem "normally"**, as you would a newspaper story or a novel.
- **Re-read the problem.** As you read the problem, **write down the important information** and the **question** the problem asks.
- As appropriate, **draw a picture, table, or graphic** of the situation, labeling with the known information.
- **Look for patterns or relationships** in the information from the problem. Write down any patterns or relationships you see.
- Use the patterns or relationships you found above to **set up an expression or equation** in terms of the information you have been given.
- **Complete your calculations** and **answer the problem's question** in a sentence.
- **Check your solution** with the original wording of the problem **to see if it "makes sense"**, both mathematically and realistically.

Math 20 Class Notes
Operations on Mixed Numbers: Sections 2.6

Operations on Mixed Number – "The Key"

- For any operation on mixed numbers (addition, subtraction, multiplication, division, etc.), we do the following:
    - First convert each mixed number to an improper fraction
    - Do the necessary operation(s), simplifying the results
    - Convert results greater than 1 back to a mixed number, if appropriate for the application involved
- An important reason we change mixed numbers to improper fractions, then do calculations is because improper fractions are much easier to use in calculations than mixed numbers.
- Use the Group Activity "Cooking for Company" together as a class example. The activity is on page 150 of the textbook.

Group Activity

Cooking for Company

Estimated time: 15 minutes

Group Size: 3

This recipe for chili serves 6 people.

CHILI

1 tablespoon oil

2 onions

$1\frac{1}{2}$ lb ground beef

14 oz canned tomatoes

$1\frac{1}{4}$ teaspoons chili powder

$\frac{3}{8}$ teaspoon cumin

$1\frac{1}{8}$ teaspoons salt

$\frac{1}{8}$ teaspoon cayenne pepper

4 whole cloves

15 oz canned kidney beans

1. Suppose that this recipe is to be used for a Super Bowl party for 30 people. Each student in the group should select three ingredients and determine the amount needed to make this recipe for 30 people. Then write the revised recipe.

   5 T oil; 10 onions; $7\frac{1}{2}$ lb beef; 70 oz tomatoes; $6\frac{1}{4}$ teaspoons chili power; $1\frac{7}{8}$ teaspoons cumin; $5\frac{5}{8}$ teaspoons salt; $\frac{5}{8}$ teaspoon cayenne pepper; 20 whole cloves, 75 oz beans

2. Suppose that this recipe is to be used for a dinner party for 9 guests. Each student in the group should select three ingredients and determine the amount needed to make this recipe for 9 people. Then write the revised recipe.

   $1\frac{1}{2}$ T oil; 3 onions; $2\frac{1}{4}$ lb beef; 21 oz tomatoes; $1\frac{7}{8}$ teaspoons chili power; $\frac{9}{16}$ teaspoon cumin; $1\frac{11}{16}$ teaspoons salt; $\frac{3}{16}$ teaspoon cayenne pepper; 6 whole cloves, $22\frac{1}{2}$ oz beans

Math 20 Class Notes
Addition and Subtraction of Fractions: Sections 3.1 – 3.3

- Definitions
  - **Like Fractions** are those with the same denominator.
  - **Unlike Fractions** are fractions with different denominators.
  - **Building Fractions** means to multiply both the numerator and the denominator by the same value, thus preserving the "value" of the fraction

- **Adding and Subtracting "Like Fractions"**
  - Use objects from the real world (fractions) to illustrate how to add or subtract like fractions
  - Addition: Add the numerators and put the result over the "common denominator"
  - Subtraction: Subtract the numerators and put the result over the "common denominator"

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c} \qquad \frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$$

- **Ordering Fractions**
  - We can now order fractions by "building fractions" to a common denominator.

- **The Least Common Denominator (LCD)**
  - The Least Common Multiple (LCM) of two (or more) numbers is the smallest number that is a multiple of the given numbers.
  - Using prime numbers and factoring trees to find the LCM of two numbers.
  - To find the best common denominator, find the LCM of the two denominators, called the LCD (**Least Common Denominator**)

- **Adding and Subtracting "Unlike Fractions"**
  - Build both fractions so the denominators are the LCD of the two denominators.
  - **Building**: $\frac{a}{b}\cdot\frac{c}{c}=\frac{ac}{bc}$. Choose the value c so that "bc" is the LCD.
  - Once you have "built" both fractions so the denominators are both the LCD, the fractions are like fractions, which you can add or subtract easily.

- **Order of Operations with Fractions – Average of fractions**
  - Demonstrate the use of Order of Operations with fractions in calculating an average (arithmetic mean) as the sum of all the data divided by the number of data values.
  - Demonstrate other applications using problems from Practice Exercises.

Math 20 Class Notes
More Operations on Mixed Numbers: Sections 3.4 & 3.5

(Review Rounding from Section 1.4)

Rounding Mixed Numbers as an Estimation Tactic

- We round mixed numbers similarly to rounding whole numbers
    - Rounding to the nearest place value (**whole numbers**)
        - To round to a specific place value:
            - If the NEXT digit is a 0,1,2,3,4 then we round "down"
            - If the NEXT digit is a 5,6,7,8,9 then we round "up"
            - We call that next digit the "Test Digit"
    - Rounding to the nearest place value (**mixed numbers**)
        - To round to the nearest whole number:
            - If the Proper Fraction portion digit is less than ½, then we round "down" to the nearest whole number
            - If the Proper Fraction portion digit is equal to or more than ½, then we round "up" to the nearest whole number

Operations on Mixed Number – "The Key"

- For **ANY** operation on mixed numbers (addition, subtraction, multiplication, division, etc.), we do the following:
    - First convert each mixed number to an improper fraction
    - Do the necessary operation(s), simplifying the results
    - Convert results greater than 1 back to a mixed number, if appropriate for the application involved
- We will NOT study tactics that require any operations on mixed numbers that require the numbers to stay in "mixed number form", including any "borrowing" tactics.
- An important reason we change mixed numbers to improper fractions, then do calculations is because improper fractions are much easier to use in calculations than mixed numbers.
- Note that Order of Operations applies to mixed numbers (in any form).
- Use Problems #36, 46, 58 on pages 206-207

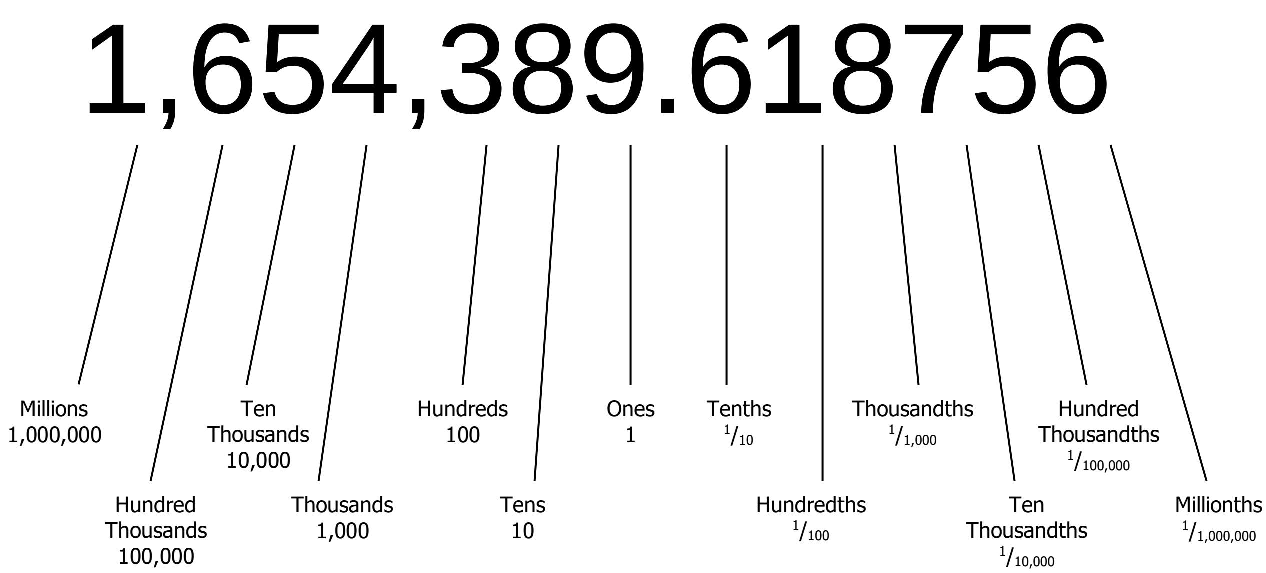
Math 20 Class Notes
Decimals Part 1 – Sections 4.1, 4.2, 4.3

(Study applications throughout this chapter)

A. Terminology

1. **Decimals** are another way to write fractions and mixed numbers. When we use decimals to write numbers, we call this **decimal notation**.
2. The **decimal point** separates the whole number portion of a value from the "fractional" portion of the number.
3. We say the number of **decimal places** is the number of digits to the right of the decimal point.
4. If a value is expressed exactly as a decimal, we say it is an **exact decimal**. If we need to do any rounding on the decimal, we say it is an **approximate decimal**.
5. The **place value name** is the number written using digits (e.g. – 892).
6. The **word name** is the number written using words (e.g. – eight hundred ninety-two). We use the word "and" to separate the whole number portion from the fractional portion, just as with mixed numbers.

B. Decimal Places and Place Values (see graphic below)



C. Writing Decimals and Operations on Decimals

1. Word Names

a. To write the word name for a decimal from the place value name, we "read" the decimal and write down what we read. Knowing the place values is essential.

2. **Place Value Names**
    a. To write the place value name of a decimal from the word name, we read the word name, writing down the place values as we go and using the word and to identify where the decimal point is written. Knowing the place values is essential.
3. **Rounding a decimal**
    a. If the digit after the place value where you wish to round (called the "test digit") is 0, 1, 2, 3, or 4, we round "down".
    b. If the digit after the place value where you wish to round is 5, 6, 7, 8, or 9, we round "up".
4. **Changing a decimal to a fraction or mixed number** is done by using the place value of the right-most decimal place of the number, then writing the fraction. You must then simplify the fraction.
5. **Addition and Subtraction**
    a. We can add and subtract decimals using a vertical format, just like with whole number addition. Just line up the decimal points before you add. We estimate the results with front rounding.
6. **Multiplication**
    a. We can multiply decimals using a vertical format. Just "pretend" the decimal are whole numbers. Then count the total number of decimal places in the problem. That is the number of decimal places in the product. We estimate the results with front rounding.
7. **Powers of 10 and 1**
    a. Multiplying by 10 moves the decimal point to the right one place. Multiplying by a power of 10 moves the decimal point the same number of places as the exponent.
    b. Multiplying by $^1/_{10}$, $^1/_{100}$, $^1/_{100}$ (powers of $^1/_{10}$ = 0.1) … moves the decimal point to the left the same number of places as the number of zeroes in the denominator.
    c. Multiplying by $^1/_{10}$, $^1/_{100}$, $^1/_{100}$ (powers of 0.1) … is the same as dividing by 10, 100, 1000 (powers of 10) … respectively.
    d. ANY power of 1 equals 1

Math 20 Class Notes
Decimals Part 2 – Sections 4.4 – 4.6

A. Division with Decimal

Demonstrate how to divide decimals using long division.

- Move the decimal point in both the divisor and the dividend so that the denominator is NOT a decimal.
- Place a decimal point for the quotient above the "new decimal point" in the dividend.
- Divide as usual.
- Round your result to the given place value.
- If no place value for rounding is given, round appropriate to the problem or continue dividing until the remainder is zero.
- We can "front round" to **estimate with long division**, but we should round both values up or round both values down to improve the estimate.

B. Changing Decimals to Fractions

We can do this by writing out the word name of the decimal, and then use the word name to write an equivalent fraction. Then simplify the fraction.

C. Changing Fractions to Decimals

Use long division (or a scientific calculator) to accomplish this.

D. Miscellaneous Topics

8. Ordering decimals is very easy, using the place values from left to right.

2. "Mixed Operations": We perform operations containing decimals and fractions by changing the numbers to the same form, then performing the operations.

3. Order of Operations: Order of Operations is the same for decimals as for Whole Numbers, Integers, and Fractions.

F. Average, Median, Mode with Decimals

1. We can **calculate an average** with decimals by adding all the numbers and dividing by the number of values added … just like with whole numbers.
2. We can **calculate the median** with decimals by ordering the decimals from least to greatest, then picking the middle value from the list (odd sample size) or averaging the middle two values from the list (even sample size).
3. We can **calculate the mode** with decimals by identifying the most frequent value in the list.

Math 20 Class Notes
Real Numbers – Part 1: Sections 10.1 – 10.3

1. **Terminology**
   a. **Positive Numbers**: These are the most commonly used numbers in our everyday lives. They are greater than zero, and are on the right side of the number line from zero.
   b. **Negative numbers**: These are the "opposites" of the positive numbers. They are less than zero, and are on the left side of the number line from zero.
   c. **Integer**: An integer is a whole number, zero, or the "**opposite**" (negative) of the whole numbers.
      i. You can find the **opposite** of an integer using the number line, by looking at the other side of the number line the same distance from zero.
      ii. You can find the **absolute value** of an integer by "removing the sign".
      iii. Comparing integers with inequality symbols < and >.
   d. **Zero**: A number that is neither positive nor negative. It is the midpoint between positive and negative numbers on the number line.
   e. **Real Numbers**: We sometimes refer to all positive and all negative numbers as real numbers.
      i. **Rational Numbers** are all numbers that can be written exactly as a fraction
      ii. **Irrational Numbers** are all numbers that cannot be written exactly as a fraction. Examples of irrational numbers are π and $\sqrt{5}$.
   f. **Absolute Value**:
      i. The distance of a number from zero on the number line.
      ii. A number with its sign (+ or –) removed.

2. **Operations with Real Numbers**
   a. You can **add** two real numbers:
      i. If the two addends have the same sign, add their absolute values and give the result the same sign as the addends.
      ii. If the two addends have different signs, subtract the smaller absolute value from the larger absolute value, then give the result the same sign as the number with the larger absolute value.
   b. Properties of addition
      i. <u>Commutative Law of Addition</u>: Two real numbers can be added in either order.
      ii. <u>Associative Law of Addition</u>: Three real numbers being added can be grouped with parentheses around the first two real numbers or the second two real numbers.
      iii. <u>Addition Property of Zero</u>: Adding zero to any number will not change the original number. We say that zero is the **additive identity**.
      iv. <u>Addition Property of Opposites</u>: Adding a real number and its opposite will yield a sum of zero. We say these two opposites are called **additive inverses** of each other.
   c. You can **subtract** two real numbers by using **the definition of subtraction:** $a - b = a + (-b)$ to change the problem to addition, then add.

Math 20 Class Notes
Real Numbers – Part 2: Sections 10.4 & 10.5

1) **Operations with Real Numbers:**
   d. You can **multiply** two integers:
      i. If the two numbers have the same sign, the result is positive.
      ii. If the two numbers have different signs, the result is negative.
   e. Properties of Multiplication
      i. Commutative Law of Multiplication: Two numbers can be multiplied in either order.
      ii. Associative Law of Multiplication: Three numbers being multiplied can be grouped with parentheses around the first two numbers or the second two integers.
      iii. Multiplication Property of Zero: Multiplying any number by zero is zero.
      iv. Multiplication Property of One: Multiplying any number by one will not change the original number.
      v. Multiplying an even number of negative numbers will be positive.
      vi. Multiplying an odd number of negative numbers will be negative, as pairing cancels out the negatives … except the last one. This also applies to an even or odd "power" of a negative number.
   f. You can **divide** two numbers:
      i. If the two numbers have the same sign, the result is positive.
      ii. If the two numbers have different signs, the result is negative.
      iii. Note that these rules are the same as for multiplication, because of the **definition of division**: $a \div b = a \cdot {}^{1}/_{b}$
   g. Properties of Division:
      i. Zero divided by any non-zero number is zero.
      ii. Any non-zero number divided by zero in "undefined".

2) **Order of Operations with Real Numbers**
   a. First do operations inside grouping symbols, like parentheses and brackets.
   b. Second do any exponents.
   c. Third do multiplication and division (left to right).
   d. Last do addition and subtraction (left to right).
   e. Note that the Order of Operations is the same for whole numbers and real numbers!
   f. This is a good time to practice!

Math 20 Class Notes
Ratios and Proportions: Chapter 5

A. Terminology

1. A **ratio** is a comparison of two measurements using division.
2. **Like measurements** are measurements that have the same units.
3. **Unlike measurements** are measurements that have the different units.

B. Rates, Unit Rates, and Unit Cost

1. A **rate** is a comparison of two unlike units using division. Or, it is a ratio of unlike measurements.
2. A **unit rate** is a rate with a denominator value of 1.
3. When writing ratios, it is important to simplify them.
4. We do not drop the units when they are unlike, but if they are like measurements with the same units, the units "cancel" as do numbers.
5. You can change a rate to a unit rate by leaving the units alone and dividing the numerator of the rate by the denominator of the rate.
6. A **unit cost** is the cost of each of a collection of items to be sold. This is a unit rate for the cost of the item.
    1. For example, if you can buy 3 pounds of apples for \$3.39, then the unit cost is the price for each pound of apples. Thus, we get the rate of ${}^{\$3.39}/_{3\text{ pounds}}$. We can reduce this rate to the unit rate of ${}^{\$1.13}/_{\text{pound}}$, which is the unit cost for the apples.

C. Proportions

1. A **proportion** is two ratios set equal to each other.
2. The **method of cross multiplication** is multiplying the numerator of one ratio by the denominator of the other to form two **cross products**. You then set the cross products equal to each other to solve the proportion.
3. **Solving a proportion** means to find the missing value, usually shown as a letter. This will yield a number that makes the proportion "true".
    a. A "true proportion" is where the cross-products are equal.
    b. A "false proportion is where the cross-products are not equal.

D. Applications of Proportions

1. Solving word problems using proportions.
    a. Always use the Problem Solving Tactics in the Syllabus to help.
    b. Write the two ratios.
    c. Form the proportion.
    d. Solve the proportion you created.
    e. Write the solution in a phrase or short sentence, including the appropriate units.
2. One important application of proportions is with **similar triangles**.

a. A pair of triangles is called **similar** if the ratio of corresponding sides is equal. In effect, this means that the shape of the triangles is the same, but the size differs.
b. An example would be two triangles where the sides of the first triangle are 3 cm, 4 cm, and 5 cm and the sides of the second triangle are 9 inches, 12 inches, and 15 inches.
c. Note that the ratios of corresponding sides are equal: $^{3\text{ cm}}/_{9\text{ in}} = {}^{4\text{ cm}}/_{12\text{ inches}} = {}^{5\text{ cm}}/_{15\text{ in}}$.

Math 20 Class Notes
Percents: Chapter 6

A. Terminology

1. A **base unit** is the denominator of a ratio, when you use ratios to compare numbers.
2. A **percent** is such a ratio where the base unit is 100. Another way to look at the word "percent" is "per one hundred" or "one hundredth".
3. **A percent that represents chance**:
   a. The percent cannot be more than 100%.
   b. The percent cannot be less than 0%.

B. Finding a Percent

1. Finding a percent comparison.
   a. Write the ratio.
   b. Write another ratio with an unknown as the numerator and 100 as the base unit.
   c. Use the two ratios to create a proportion.
   d. Solve the proportion by cross-multiplying.
   e. The solved for unknown is the unknown percent.

C. Changing Between Percents and Decimals

1. Change a percent to a decimal by moving the decimal point two places left.
2. Change a decimal to a percent by moving the decimal point two places right.

D. Changing Between Percents and Fractions

1. Change a percent to a fraction by changing it to a decimal first using the word name, then to a fraction.
2. Change a fraction to a percent by using long division to change it to a decimal first, then to a percent.

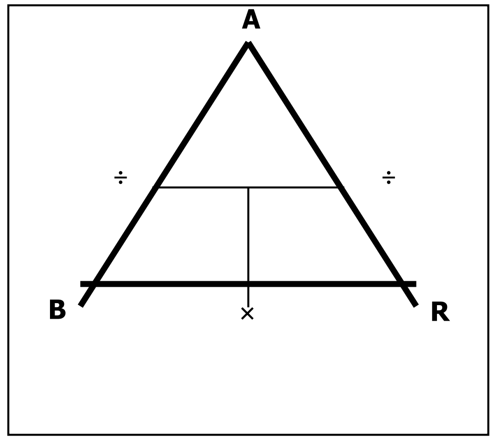
E. Using Proportions to Solve Percent Problems

1. Set up the proportion, $^{\text{Amount}}/_{\text{Base}} = {}^{P}/_{100}$, then solve the proportion for the unknown.

F. Basic Percent Problem

1. In a **basic percent problem** (R% of B is A), we define the following vocabulary:
   a. The **rate** (R) is the percent.
   b. The **base** (B) is the value the percent is applied towards. It follows the "of".
   c. The **amount** (A) is the result of applying a percent. It is the amount compared to the base B.

G. Solving a Basic Percent Problem



1. Using the "triangle" graphic.
   a. Draw the graphic.
   b. Cover the unknown value.
   c. Reading from the top or the right, do what the triangle shows to solve the problem.
   d. Check your work and write out the result in a sentence.
2. Using formulas.
   a. Identify the rate (R), the base (B), and the amount (A).
   b. Use the appropriate formula to solve for the unknown value:
      i. $\mathbf{R = A \div B}$
      ii. $\mathbf{B = A \div R}$
      iii. $\mathbf{A = R \times B}$
   c. Check your work and write out the result in a sentence.

H. Applications of Percents

1. **Percent Increase**: If the amount (A) is an "increase" over the base (B), then the percent is called a "percent of increase".
   a. Determine the "amount of increase"
   b. Use this to determine the percent of increase.
2. **Percent Decrease**: If the amount (A) is a "decrease" over the base (B), then the percent is called a "percent of decrease".
   a. Determine the "amount of decrease"
   b. Use this to determine the percent of decrease.

Math 20 Class Notes
Measurements and Conversions: Chapter 7

A. Definitions

1. A **measurement** is a value found using a tool that describes something in the real world.
2. A **unit of measure** is the "label" used in a measurement to describe the measurement. The basic kinds of units that all others are based on are length, time, and mass.
3. **Equivalent measurements** are those that represent the same amount, though they may have different units of measure.
4. **Unit fractions** are fractions whose numerator and denominator are equivalent measurements. We use unit fractions to change from one unit of measure to another.
5. The **English System** is the measurement system commonly used in the United States. Basic units include the pound, the second, and the foot.
6. The **Metric System** (Systeme Internationale or SI) is the measurement system used by most of the world, including most US scientists.
7. Units for **length**, **capacity** (volume), **weight/mass**, and **temperature** will be studied.

B. Conversion Basics (using the English System)

1. Create a **unit fraction** from a known conversion factor.
    a. Find a **conversion factor** that includes both the "unit you have" (in the measurement) and the "unit you want".
    b. Choose the denominator for your unit fraction to be the unit you have – "Put what you don't want in the basement". The numerator is the unit you want to change towards.
2. Multiply or divide your measurement by the unit fraction.
    a. Remember, units "cancel" just like numbers!
    b. **Multiply the numerators, divide the denominators, ignore the ones**
3. Check you answer to see if it makes sense.
4. NOTE: When converting in units between systems of measurement, the conversion factors are often approximate.
5. Using "multi-step conversions" is done by "stringing" multiple unit fractions, one for each step. Use a "path" tactic for planning the steps.

C. Conversions Within the Metric System – Metric Prefixes

1. Discuss the common metric prefixes including: milli, centi, deci, deka, hecto, kilo, mega, …
2. Conversions from one metric unit to the same unit with a different metric prefix is about multiplying/dividing by a power of 10.

D. Conversions Using Formulas

1. Discuss conversions between Celsius and Fahrenheit temperature.
    a. $\mathbf{F = {}^{9}/_{5}C + 32}$ (Celsius to Fahrenheit)
    b. $\mathbf{C = {}^{5}/_{9}(F - 32)}$ (Fahrenheit to Celsius)

Math 20 Class Notes
Conversion Tables and Formulas

Units of Length

American
12 inches (in) = 1 foot (ft)
3 ft = 1 yard (yd)
36 in = 1 yd
5280 ft = 1 mile

Metric
1 kilometer (km) = 1000 meters (m)
1 hectometer (hm) = 100 m
1 dekameter (dam) = 10 m
1 decimeter (dm) = $^{1}/_{10}$ m
1 centimeter (cm) = $^{1}/_{100}$ m
1 millimeter (mm) = $^{1}/_{1000}$ m

American $\leftrightarrow$ Metric

1 in = 2.54 cm	1 cm ≈ 0.39 in
1 ft ≈ 0.30 m	1 m ≈ 3.28 ft
1 yd ≈ 0.91 m	1 m ≈ 1.09 yd
1 mi ≈ 1.61 km	1 km ≈ 0.62 mi

Units of Weight or Mass

American
16 ounces (oz) = 1 pound (lb)
2000 lb = 1 ton

Metric
1 kilogram (kg) = 1000 grams (g)
1 hectogram (hg) = 100 g
1 dekagram (dag) = 10 g
1 decigram (dg) = $^{1}/_{10}$ g
1 centigram (cg) = $^{1}/_{100}$ g
1 milligram (mg) = $^{1}/_{1000}$ g

American $\leftrightarrow$ Metric

1 oz ≈ 28.35 g	1 g ≈ 0.035 oz
1 lb ≈ 0.45 kg	1 kg ≈ 2.20 lb

Units of Capacity

American
1 cup (c) = 8 fluid ounces (fl oz)
1 quart (qt) = 2 pints (pt)
1 pt = 2 c
1 gallon (gal) = 4 qts

Metric
1 kiloliter (kL) = 1000 liters (mL)
1 hectoliter (hL) = 100 L
1 dekaliter (daL) = 10 L
1 deciliter (dL) = $^{1}/_{10}$ L
1 centiliter (cL) = $^{1}/_{100}$ L
1 milliliter (mL) = $^{1}/_{1000}$ L

American $\leftrightarrow$ Metric

1 fl oz ≈ 29.57 mL	1 L ≈ 0.39 fl oz
1 pt ≈ 0.47 L	1 L ≈ 2.11 pt
1 qt ≈ 0.95 L	1 L ≈ 1.06 qt
1 gal ≈ 3.79 L	1 L ≈ 0.264 gal

Temperature

F → C: $C = \frac{5}{9}(F - 32)$

C → F: $F = \frac{9}{5}C + 32$

Math 20 Class Notes
Introduction to Statistics: Sections 9.1 & 9.4

- **Graphs and Measures of Central Tendency**
  - Graphs as a Visual Summary
    - Constructing and interpreting **bar graphs**
      - Bar graphs are good for showing the **relative sizes** of categories
        - For example, a bar graph of tablet manufacturers by sales can show the relative sales for each manufacturer for comparing sales.
    - Constructing and interpreting **line graphs**
      - Line graphs are good for **showing trends over time**
        - For example, a line graph showing the mean monthly temperature for a chosen location can show the trends of temperature over time.
    - Constructing and interpreting **pictographs**
      - Pictographs can be good for comparing the relative size of categories, similar to bar graphs. However, the use of symbols (pictographs) instead of bars does make the graph understandable to a more "elementary" audience.

- **Mean, Median, and Mode**
    - **Mean: The arithmetic average**
      - Add up the data values, then divide the sum by the number of data values.
    - **Median: The middle value in an ordered list to data values**
      - If the sample size is odd, order the data values and choose the middle value in that list
      - If the sample size is even, order the data values. The median is the mean of the middle two data values
    - **Mode: The most frequent value**
      - It is possible to not have a mode. This requires all the data values to occur at the same frequency.
      - It is possible to have more than one mode. If two (or more) data values occur at the highest frequency in the sample, then there is more than one mode.
      - Samples with 2 modes are called "bimodal". Samples with 3 modes are called "trimodal", and so forth.