COMPASS Placement Test

Review Packet

For preparing to take the COMPASS Placement Test

Portland Community College
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The COMPASS Test

The COMPASS placement test is offered in Reading, Writing, and Math. The test helps to determine whether you have the knowledge to succeed in the classes you are planning to take or whether taking some preparatory classes will ensure your success. Taking the three tests separately is usually helpful to ensure best results in all three tests.

The COMPASS test is a self-adjusting, multiple choice test that is taken at the computer. The answer to your current question will determine the next question; it will stop once it has determined your level. Consequently the test is untimed and has a different number of questions for each student. It also means that you will see questions that you don’t know, because the test will ask you more and more difficult questions until it has found something that you don’t know. Just do your best you can for each question the test presents to you.

You will receive paper, pencil, and a calculator. Since you will work on the computer you will not be allowed to bring food or drink. Once you have completed a test you will receive a printout with your scores and a recommendation of classes to take. You should then make an appointment with an advisor to discuss your course work.
The COMPASS test can be taken on all three campuses (Cascade, Rock Creek, and Sylvania) and the Southeast Center. Following are the locations and phone numbers.
Contact the testing center of your choice for opening hours. During those opening hours you can take the test at any time.

- Cascade: SSB 204, 503-978-5234
- Rock Creek: Bldg. 9, Rm. 118, 503-614-7300
- Southeast Center: Mt Tabor Hall, Room 107 & 109, 503-788-6277
- Sylvania: CC 216, 503-977-4533
- Testing Centers website: http://www.pcc.edu/resources/testing/

After you have taken the placement test in the three subjects of Reading, Writing, and Math, you should see an advisor to discuss your course work. Here are the advising centers for each campus:

- Cascade: SSB 150, 503-978-5271, opening hours:
  - Monday, Tuesday & Thursday: 8:30am - 6:00pm
  - Wednesday: 8:30am - 7:00pm
  - Friday: 8:30am - 4:00pm

- Rock Creek: Bldg. 9, Rm. 102, 503-614-7297, opening hours:
  - Monday to Thursday: 8:30am - 6pm
  - Friday: 8:30am - 4pm

- Southeast Center: Mt Tabor Hall, Room 152, 503-788-6240, opening hours:
  - Monday to Thursday: 9am - 6pm
  - Friday: 8am - 4pm

- Sylvania: CC 216, 503-977-4531, opening hours:
  - Monday, Tuesday and Thursday: 8am - 6pm
  - Wednesday: 8am - 7pm
  - Friday: 8am - 5pm

- Academic Advising website: http://www.pcc.edu/resources/advising/
Test-taking Strategies

1. **Take the Placement Test Seriously**
   Giving your best during the test can save you several terms of math, reading, and writing classes, and therefore a lot of time and money. What you don’t know, you don’t know. That’s fine. But if you know something, make sure you show it on the test so that you are placed into the appropriate class for your skill level.

2. **Prepare For the Test**
   It is important that you review your knowledge before you take the test, particularly if you have not been in school for many years. Go over the following parts in this review packet to refresh your memory about the things you once knew. This packet is not designed to help you learn material that you never knew. For that you should take a class.

   It is equally important, however, that you are physically prepared for the test. Be sure to get enough sleep the night before, and eat something nutritious before arriving for the test. Don’t consume anything with caffeine or a lot of sugar right before the test. Caffeine might make you feel more jittery and less patient, causing you to skip important steps. Too much sugar will give you a short energy boost followed by a sense of fatigue. Drink water or tea instead.

3. **Take Time**
   The Compass Test is not timed which means that you can take as much time as you need. Make use of that! Read the questions carefully, think about them, do your work on paper, and then choose an answer. Your score does not depend on how long you take for each question. Your score *only* depends on whether you choose the right answer.

4. **Read the Questions Carefully**
   Don’t assume anything. Follow the instructions of the question exactly. Read all the details very carefully. A simple “not” can change everything around. It helps to copy the question onto paper and underline the important information or rewrite it in your own words.
5. **Work Math Problems out on Paper**  
Since the COMPASS Test is a test that you take on the computer make sure to copy math problems onto paper and work them step by step. It’s worth it! Working a problem out carefully and minding all the details gets you the points to place you in the right class.

6. **Take a Break**  
You can take a break whenever you like! Just go to the testing supervisor, and s/he will save your work. You can continue when you come back. You can even come back the next day.

This is very important because in order to do well on the test you need to concentrate. So if you need to use the restroom, go. If you are thirsty or hungry, go drink and eat. If you are tired, get up and take a walk or go home and come back the next day.

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**Additional Resources for the COMPASS placement test**

- The testing centers website: [http://www.pcc.edu/resources/testing/](http://www.pcc.edu/resources/testing/)
- The testing centers website for the COMPASS placement test: [http://www.pcc.edu/resources/testing/college-placement.html](http://www.pcc.edu/resources/testing/college-placement.html)

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**Good Luck!**
The COMPASS Reading Placement Test tests the ability to refer and reason based on the information presented. The following is a review of topics that may be found on this test.

I. Finding the Main Idea 8
II. Vocabulary: Word Meaning and Context 9
III. Supporting Details 11
IV. Inferences 12
V. Implied Main Ideas and Central Points 14
I. Finding the Main Idea

- The main idea is the author’s main point.
- The author’s main point is a general statement supported by specific details.
- Ask “what is the point being made about the topic?” to find the main idea.
- The main idea can be located at the beginning, middle, or end of a paragraph.

Practice Exercises

Exercise 1:
(1) For over a century, marketers have been using packaging to get consumers to buy their products. (2) Since the 1890s, Quaker Oats has put its rolled oats in a box with a picture of a religious Quaker. (3) The package suggests that their cereal is wholesome. (4) In 1898, a man named C.W. Post wanted to create the image of a healthy breakfast food. (5) So he placed a copy of the pamphlet “The Road to Wellville” in each box of Grape Nuts cereal. (6) Packaging is so important that store brands often try to take advantage of it by looking like the bestselling national brands. (7) Their packages imitate the color of the labels, the shape of the container, and so on.

What is the topic?
Cereal  Packaging  Products

Which sentence is the main idea of the passage?
1  4  6

Exercise 2:
(1) Many of our wedding customs reflect the fact that women once had little say in who their groom would be. (2) Before the twelfth century, the best man was a warrior friend who helped a man capture and kidnap a woman he desired, often from another tribe. (3) Carrying the bride over the threshold isn’t simply a romantic act. (4) Originally, it represented the kidnapping of the daughter who would not willingly leave her father’s house. (5) After a man captured or bought a bride, he disappeared with her for a while on a honeymoon, so that her family couldn’t rescue her. (6) By the time they found the couple, the bride would already be pregnant.
What is the topic?
Grooms  Wedding Customs  Kidnapping

More practice exercises can be found on Townsend Press Online Learning Center:
http://www.townsendpress.net/home.php

II. Word Meaning

Context clues used to find word meaning:

- the surrounding text
- examples of the new word
- synonyms: a word that means the same as another word
- antonyms: a word opposite in meaning to another word

Practice Exercises

Directions for Exercises 3 and 4: Using the surrounding text, choose the meaning of the bold-faced word.

Exercise 3:
On an impulse, Carla bought a lot of merchandise at the store’s “going out of business” sale. She later regretted that she hadn’t planned her purchases more carefully.

1. dare  2. review  3. sudden wish or urge

Exercise 4:
“Mr. Allen,” said the instructor, “please cease cracking your gum in class.”

1. enjoy  2. begin  3. stop

Directions for Exercises 5 and 6: Using the examples in the following sentences, choose the meaning of the bold-faced word.

Exercise 5:
Assets such as good health, a loving family, and an enjoyable job make life rewarding.

Assets are:

1. things of value  2. rewards on the job  3. helpful people
Exercise 6:
Newspaper reporters have been fired for fictitious reporting that included quotations which were never said and events that never occurred.
   fictitious means:
   1. true-life 2. unknown 3. not real

Directions for Exercises 7 and 8: Underline the synonym of the bold-faced word in the following sentences.

Exercise 7:
Hal was a mediocre student. He was an average baseball player as well.

Exercise 8:
Students are often apprehensive of final exams, but with the right study skills, they don’t have to be fearful.

Directions for Exercises 9 and 10: In the following sentences, first underline the antonym of the bold-faced word then choose the meaning of the word in bold.

Exercise 9:
I thought it was difficult to ascend the mountain, but I discovered that climbing down it was even worse.
   1. climb up 2. walk around 3. climb down

Exercise 10:
The teacher commended two students on the outstanding work they were doing. Then he criticized the rest of the class for doing so poorly.
   1. blamed 2. graded 3. praised

More practice exercises can be found on Townsend Press Online Learning Center: http://www.townsendpress.net/home.php
III. Supporting Details

- Supporting details are facts that support the main idea.
- Major details explain and develop the main idea.
- Minor details develop and make clear major details.

Tips:
- Addition words such as “moreover”, “another”, etc., signal major details.
- List words such as “a number of,” “a few reasons,” etc., tell you that a list is coming.

Practice Exercises

Exercise 11:
(1) Some countries require warnings on cigarette packs much stronger than the warnings in the United States. (2) New warnings adopted by Australia now show pictures of cancerous lungs and bloodied brains with text that cautions, "Smoking doubles your risk of stroke.” (3) England for years has had packages that feature large, blunt messages, such as "Smoking kills." (4) The messages span at least a third of the box in large lettering. (5) In Canada, a pack of smokes comes with stark labels of decayed teeth.

What is the main idea?
1. Sentence 2 2. Sentence 5 3. Sentence 1

How many major details are in this passage?
1. three 2. four 3. five

Exercise 12:
(1) According to researchers, our personalities are reflected in the ways we shop. (2) Organized shoppers are efficient people who go to the store armed with lists and clear ideas of what they want and how to get it quickly. (3) Impulsive shoppers are people who buy something because they see it and like it. (4) Anxious shoppers are fearful people for whom shopping means having to make decisions, something they hate to do. (5) Anti-shoppers are disinterested people who don’t care about possessions.
How many major details are in this paragraph?

1. one
2. two
3. three
4. four
5. five

Exercise 13:
Identify the topic, main idea, and supporting details from the sentences below.

1. The spinal column prevents the body from caving in on itself.
2. The spinal column protects the delicate nerve tissue of the spinal cord.
3. Roles of the spinal column.
4. The human body could not function without the spinal column.

Topic: ____
Main Idea: ____
Supporting Details: ____

More practice exercises can be found on Townsend Press Online Learning Center: http://www.townsendpress.net/home.php

IV. Inferences

- Inferences should be based on the facts presented.
- Conclusions drawn from inferences should be based on evidence, information, experiences, and common sense.

Practice Exercises

Directions for Exercises 14-17: Circle the letter of the inference that is based on the facts presented in each passage below.

Exercise 14:
“Do you believe in life after death?” the boss asked one of his employees.
“Yes, sir,” the new employee replied.
“Well, then, that makes everything just fine . . .” the boss went on. “After you left work early yesterday to go to your grandmother’s funeral, she stopped in to see you.”

What can be concluded from the information presented in the passage above?

a. The dead grandmother’s ghost came to the office looking for her grandson.
b. The boss has a wild imagination.
c. The employee had lied about going to his grandmother’s funeral.

**Exercise 15:**
Research has shown that the average mother cradles her baby in her left arm. The position allows the infant’s head to be near the comforting sound of the mother’s heartbeat. Interestingly, this form of cradling has been noted worldwide among women in modern societies, in traditional villages, and in art depicting motherhood in centuries past.

The information in this passage shows that:

a. Only mothers who are taught to cradle the baby on the left know they should do so.
b. Mothers instinctively know how to cradle their babies in a comforting way.
c. Fathers possess no instinctive skills to take care of children.

**Exercise 16:**
Fast foods tend to be high in calories and saturated fat. People who eat a lot of fast food are at greater risk for obesity. They’re also more likely to develop diabetes.

This passage shows that:

a. Consumption of fast foods is linked to health problems.
b. People who eat fast foods become obese.
c. All fats are harmful.

**Exercise 17:**
Most animals don’t eat moss. It is hard to digest, and it has little nutritional value. But reindeer fill up with lots of moss. Why? The moss contains a special chemical that helps reindeer keep their body fluids warm.

Reindeer eat moss because:

a. They like the taste.
b. They live where it is very cold.
c. It is the only food available.

More practice exercises can be found on Townsend Press Online Learning Center: [http://www.townsendpress.net/home.php](http://www.townsendpress.net/home.php)
V. Implied Main Ideas and Central Points

- Main ideas may be suggested or implied. In cases such as these, the main idea may be inferred from the supporting details provided.
- Passages with more than one paragraph often have a “central point.” The central point may be stated or implied.
- To find the central point, look for the topic of the passage and consider the supporting details provided within the passage.

Practice Exercises

Directions for Exercises 18-20: For each passage, circle the letter of the sentence that is the implied main idea.

Exercise 18:
It was reported that in 1711, when work on St. Paul’s Cathedral in London was completed and shown to King George I, he exclaimed the building was “aweful” and “artificial.” Its architect, Christopher Wren, took the king’s judgment as a great compliment. In the 1700s, “aweful” meant “awe-inspiring,” and “artificial” meant “full of great artistry.”

a. The meaning of words can change dramatically over the course of time.
b. St. Paul’s Cathedral was designed by architect Christopher Wren.
c. King George I was known to criticize people harshly.

Exercise 19:
In 1957, one researcher decided to investigate the power of subliminal advertising. At a movie theater, he inserted single-frame Coca-Cola and popcorn ads among the 26 frames projected on the movie screen every second. The appearance of the ads was so rapid that the audience was unaware of seeing them. But the researcher concluded that people’s subconscious minds perceived the ads. His evidence? Coca-Cola sales increased 18 percent and popcorn sales 57.9 percent after showings of the films with subliminal ads.

a. People enjoy drinking Coca-Cola and eating popcorn while at the movies.
b. Subliminal advertising is more effective on movie screens than on TV sets.
c. Subliminal advertising appears to influence some people’s behavior.
Exercise 20:
In a tongue-in-cheek analysis of bureaucracies, Laurence Peter proposed what has become known as the Peter Principle: bureaucracy employees are promoted to their level of incompetence. People who perform well are promoted. If they again perform well, they’re promoted again. This process continues until they’re promoted to a level at which they no longer handle their responsibilities well; this is their level of incompetence. But if the Peter Principle were fully true, bureaucracies would be staffed entirely by incompetents and would function poorly. In reality, many bureaucracies function efficiently and smoothly.

a. The Peter Principle is only partly true.
b. Most bureaucracies are run by incompetents.
c. Promotions tend to be unfair.

Directions for Exercises 21-23: In each of the exercises, choose the number that expresses the central point of the following passages.

Exercise 21:
(1) Fast-food chains serve billions of hamburgers each year to customers across North America, Europe, and Japan. (2) This enormous appetite for beef creates a massive need for cattle. (3) In response, beef suppliers in Brazil, Costa Rica, and other Latin American countries are devoting more and more national land to cattle ranches. (4) Unlike cattle in the United States, Latin American cattle graze on grass. (5) This diet produces the lean meat demanded by fast-food corporations and restaurants. (6) However, a grass diet also requires a great deal of land for grazing. (7) What is the source of this extra grazing space? (8) Unfortunately, ranchers are using forest land. (9) Each year thousands of square miles of old forests are cleared to make room for cattle grazing. (10) These forests play a major role in maintaining the earth’s atmosphere. (11) In other words, the desire for red meat has helped destroy invaluable forests and created an environmental problem that is impacting the entire planet.

Which sentence expresses the central point? ________

Exercise 22:
(1) It is not surprising that people get so worked up arguing over the true nature of wolves. (2) A look at stories about wolves shows that these two contrasting opinions—that wolves are wonderful creatures and that they are vicious, dangerous beasts—have been around for many years. (3) In 1905, the American author Jack London wrote a novel called White Fang. (4) In that story, the title character is a wolf who, ultimately, lives peacefully with
human beings and even saves a man’s life. (5) Native American legends and stories show a similar positive view of wolves. (6) Native Americans lived in harmony with these creatures, and they portrayed them in stories as their respected brothers. (7) Roman mythology also tells of a bond between wolves and human beings. (8) According to legend, an infant named Romulus, after being thrown out to die, was fed by a wolf; he survived and went on to found the city of Rome in 753 B.C.

(9) On the other hand, there are all sorts of stories and legends that promote the idea that wolves are dangerous, frightening creatures. (10) Stories of werewolves—men who turn into wolves and tear people apart—were common in Europe by the 1400s and are still believed in certain remote areas. (11) In Bram Stoker’s novel Dracula, the hero is terrified when a pack of snarling wolves surround the carriage that is carrying him to Count Dracula’s castle. (12) Even in children’s literature, wolves are portrayed as evil creatures. (13) In “Little Red Riding Hood,” for example, the villain is a wolf who devours Red Riding Hood’s grandmother as well as the little girl.

Which sentence expresses the central point? ________

Exercise 23:
1) In recent years, DNA evidence has been used to free many prisoners who were convicted of crimes they did not commit. (2) Many of these prisoners were originally linked to their alleged crimes through the use of fingerprint evidence. (3) While fingerprint evidence can be useful in solving a crime, it is not as foolproof as some people think.

(4) Every individual in the world has a unique set of fingerprints, so it seems as though it should be a good tool for identification. (5) The problem arises in the difficulty of determining whether or not two sets of prints are an exact match. (6) For a set of fingerprints to be properly recorded, the fingers are inked and then rolled, one finger at a time, onto a flat surface. (7) Or else they are scanned into a machine that captures and stores each finger as a digital image. (8) Such properly recorded prints can be quickly analyzed by a computer, accurately identified, and matched with another set of properly recorded prints.

(9) Fingerprints found at a crime scene, however, are rarely clean and distinct. (10) A fingerprint at a crime scene frequently shows only about 20 percent of the whole fingertip. (11) And crime-scene prints are often distorted because they are on an uneven surface or, sometimes, have been covered in blood. (12) These partial prints are difficult to read. (13) A computer cannot make an accurate match from such a print.

(14) Therefore, it is up to fingerprint experts to compare the crime scene print with a properly recorded print. (15) A fingerprint expert must then determine, to the best of his or her ability, whether or not the prints match. (16) Frequently, the link between a crime-scene fingerprint and a particular individual is determined solely by
the skill and judgment of the person attempting to make the match. (17) Far from being foolproof, the determination is an inexact and subjective decision.

Which sentence expresses the central point? ________

More practice exercises can be found on Townsend Press Online Learning Center: http://www.townsendpress.net/home.php

**Answer Key for Reading Practice Exercises**

<table>
<thead>
<tr>
<th>Finding the Main Idea</th>
<th>Inferences</th>
<th>Inferences</th>
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<tbody>
<tr>
<td>1. Packaging, 1</td>
<td>14. c</td>
<td>15. b</td>
</tr>
<tr>
<td>2. Wedding Customs, 1</td>
<td>16. a</td>
<td>17. b</td>
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</table>

<table>
<thead>
<tr>
<th>Word Meaning</th>
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<tbody>
<tr>
<td>3. 3</td>
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<td>4. 3</td>
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<td>5. 1</td>
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<td>6. 3</td>
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<td>7. average</td>
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<td>8. fearful</td>
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<td>9. climb up, climbing down</td>
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<tr>
<td>10. praised, criticized</td>
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<th>Implied Main Ideas and Central Points</th>
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<td>12.</td>
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<tr>
<td>13. Topic: 3; Main Idea: 4; SD: 1,2</td>
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<table>
<thead>
<tr>
<th>Supporting Details: Major and Minor</th>
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<tbody>
<tr>
<td>11. 3, three (2, 3, 5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. four (sentences 2, 3, 4, 5)</td>
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<td></td>
</tr>
<tr>
<td>13. Topic: 3; Main Idea: 4; SD: 1,2</td>
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Practice exercises taken from: Townsend Press Learning Center: http://www.townsendpress.net/home.php

Consulted Texts:
Groundwork for College Reading, 2008, John Langan
Ten Steps to Building College Reading Skills, 2005, John Langan
Ten Steps to Advancing College Reading Skills, 2004, John Langan
Online Reading Resources

  This section of the BBC website is geared towards nonnative English speakers. Current and past news articles with relevant vocabulary are offered.

  A website that offers headline news in areas such as politics, entertainment, travel, business, U.S. and domestic news.

  This website offers reading practice along with exercises that focus on finding the main idea, inference, and details. Each reading passage includes an exercise for relevant vocabulary.

- Study Guides and Strategies: [http://www.studygs.net/](http://www.studygs.net/)
  This study skills website offers study skills and strategies for reading critically, speed and comprehension, reading essays and fiction, prereading material, reading difficult material, and more.

- Townsend Press Learning Center: [http://www.townsendpress.net/home.php](http://www.townsendpress.net/home.php)
  This website offers online exercises to supplement *Groundwork for College Reading* and *Ten Steps to Building/Advancing/Improving College Reading*. Textbooks are not needed to do these lessons.

- Word Surfing: [http://www.wordsurfing.co.uk/20.html](http://www.wordsurfing.co.uk/20.html)
The COMPASS Writing Placement Test tests basic knowledge on punctuation, grammar, sentence, structure, and rhetorical skills. Brief explanations and practice exercises in these areas are provided for review.

I. Punctuation 20
II. Basic Grammar and Usage 25
III. Sentence Structure 38
IV. Rhetorical Skills: Writing Strategies, Organization, and Style 38
I. Punctuation

Directions for all Punctuation Exercises: Complete the practice exercises after each punctuation explanation.

Comma

A comma is used after an introductory dependent clause (a clause at the beginning of a sentence that cannot stand alone).

Practice Exercises
1. While we were watching for Mom we started our homework.
2. Although she is wearing red blue is her favorite color.

A comma is used in a compound sentence between independent clauses (complete thoughts) joined by coordinating conjunction FANBOYS: For, And, Nor, But, Or, Yet, So.

Practice Exercises
3. The new cars arrived yesterday and they will be delivered tomorrow.
4. Most of the lecture was interesting but some students were not impressed.

Commas are used to separate words, phrases, or clauses in a series.

Practice Exercises
5. Mr. Jensen wanted us to bring apples milk and bread.
6. Rehearsals are held before school during recess and at night.
7. The supervisor wanted to know who had broken into the store why they had done so and what had been taken.

A comma is used to set off introductory words, introductory participial or infinitive phrases, and long (generally 5 words or more) introductory prepositional phrases.

Practice Exercises
8. Incidentally I was not late this morning.
9. Hoping she had enough money she approached the register.
10. To arrive on time we must leave here by six.
11. In light of our financial condition building a house is not wise.
Parenthetical statements (words, phrases, or clauses) are set off by commas. They are often words of direct address.

**Practice Exercises**
12. The quality of the material, however, was beyond question.
13. Mr. Reyerson is, I think, a very successful salesman.
14. Jamie, please clean your room.

Two adjectives modifying the same noun should be separated by a comma if the word **and** could be used between the adjectives.

**Practice Exercises**
15. Everyone envied her dark green car.
16. His arrogant condescending manner annoyed us.

A nonessential phrase or clause is set off by commas.

**Practice Exercises**
17. My father who was born in southern Utah is 83 years old.
18. Officer Jones chasing after the thief grabbed the gun.
19. Lacrosse a sport played hundreds of years ago is still popular.

An *appositive* (a word that renames the noun) is set off by commas.

**Practice Exercises**
20. Nick Van Exel the player from the Lakers looks like a camel.
21. The award was given to Jan Carol’s daughter.
22. American athletes Karl Malone and Michael Jordan are well-known to everyone.

Use a comma when it is necessary to make the meaning of a sentence clear.

**Practice Exercises**
23. Ever since our supplies have been stored in a warehouse.
24. Before the rainstorm everything was dry; after everything looked bright and green.

Use commas to separate cities and states and days and years; also use commas around direct quotations.
Practice Exercises
25. We moved here from Ann Arbor Michigan three months ago.
26. Mrs. Bitters the next meeting will be on Tuesday August 13 2001 in Seattle.
27. “Scientific research” she said “has found an unhealthy diet increases your risk of cancer.”

NO COMMAS needed to set off essential phrases or clauses.
» My sister who lives in Massachusetts is coming to visit.
   (Because the writer has more than one sister, “who lives in Massachusetts” is necessary to identify which sister is being referred to—no commas)
» One of the policemen chasing after the thief grabbed the gun. (Participial phrase “chasing after the thief” is necessary to identify which policeman—no commas)
» The committee reached a decision that was not popular. (Adjective clauses which begin with “that” are essential—no commas)

Semicolon

Use a semicolon to connect two independent clauses when the two clauses are related.

Practice Exercises
1. I can’t eat strawberries they give me a rash.
2. The soccer game begins at 2 o’clock please be on time.
3. Karen and Tom went to Clancy’s for dinner they both enjoy eating seafood.
4. David left work early he had a dentist appointment at 3:00 P.M.
5. Kate always wears earmuffs during the winter her ears are extra sensitive to the cold.

Use a semicolon to connect two independent clauses when the second clause begins with a conjunctive adverb (however, therefore, furthermore, moreover, etc.).

Practice Exercises
6. I can build a tree house with scrap lumber however, I must buy a rope ladder.
7. Ranchers put up barbed wire fences otherwise, cattle will roam to other ranches.
8. Exercising daily takes hard work nevertheless, millions of Americans do it.
9. The runner was shaking from fatigue however, she finished the marathon.
10. Walking home takes an hour furthermore, there is no sidewalk.
Use a semicolon to join a series of items that include commas.

Practice Exercises
11. I am currently taking Spanish, which I enjoy math, which I find difficult and psychology, which is my favorite subject.
12. We visited Lima, Peru Rio Dijonero, Brazil and Salt Lake City, Utah.
14. Her favorite players are Coby Bryant, a point guard Greg Oden, a center and Brandon Roy, a forward.
15. The teenager attempted to purchase beer from the undercover officer on Friday, July 4th Saturday, July 5th and Sunday, July 6th.

Colon

A colon should be used to join to independent clauses where the second clause is to be emphasized.

Practice Exercises
1. Road construction in Portland has hindered travel around town parts of Broadway, Fifth, and Sixth are closed during the construction.
2. Brian was never out of his family’s reach he carried is cell phone wherever he went.
3. Mike could not remember that the accident had totaled his car and put him in the hospital, but he was sure of one thing he was lucky to be alive.

Use a colon after an independent clause that is followed by a list of items, direct quotation, or an appositive (a word or phrase that renames another noun).

Practice Exercises
4. Julie went to the store for some groceries milk, coffee, cereal, and fruit.
5. I know the perfect job for her a politician.
6. In his Gettysburg Address, Abraham Lincoln urges Americans to rededicate themselves to the unfinished work of the deceased soldiers "It is for us the living rather to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us — that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion — that we here highly resolve that these dead shall not have died in vain, that this nation under God shall have a new birth of freedom, and that government of the people, by the people, for the people shall not perish from the earth."
Parentheses

Use parentheses to de-emphasize content.

Practice Exercises
1. The movie which is rated-R has violence and adult language.
2. Her husband says that there is no better way to spend a Sunday afternoon than eating spaghetti his favorite food and watching a movie.
3. She loved the color gray of her new car.

Dash

Use a dash to emphasize content within the dash or the content that follows the dash.

Practice Exercises
1. It was his humor not his good looks that made him so popular.
2. There are two reasons the couple are having problems money and communication.
3. She had so many errands to do go to the store, stop by the bank, and pick-up the kids that she did not know where to begin.

Use a dash to highlight an appositive that includes commas.

Practice Exercises
4. The three sisters Ally, Jessica, and Kirstin fought all the time.
5. There are three types of fruit oranges, papaya, and banana that I like to eat all the time.

Quotations

Use quotations to enclose direct quotes, to highlight a word, poems, song titles, short stories, magazine articles, newspaper articles, and chapter titles.

Practice Exercises
1. She asked, When will you be leaving for the airport? I answered, At 6:30 tomorrow morning.
2. Of Human Bondage, by Somerset Maugham
3. History is stained with blood spilled in the name of justice.
II. Basic Grammar and Usage

Articles

Types of Articles
- Definite: the
- Indefinite: a, an

Using Definite Article the
- use before singular and plural nouns when the noun is specific and/or definite
- used when referring to a particular member of a group
- use with noncountable nouns that are made more specific by a limiting or modifying phrase or clause, i.e., The tea in my cup is too cold.
- use when referring to something unique, i.e., the Pentagon
- do not use the when referring to noncountable nouns in the general sense.
  Example: The English is his second language (In this case, the use of the would be incorrect.)
- use the with noncountable nouns that are made more specific by a limiting modifying phrase or clause
  Example: The coffee in my cup is too hot to drink.
- use after the first reference to a thing
  Example: “I got a new car. The car is gray.”

Using Indefinite Articles a & an
- use to refer to any member of a group
- use with singular nouns when the noun is general
- use the indefinite quantity world “some” for general plural nouns

Rules for using a & an
- a + singular nouns beginning with a consonant
- an + singular nouns beginning with a vowel
- a + singular noun starting with a consonant sound
  Example: “a user”
- some + plural noun
Using *a* & *an* when noun is modified by an adjective
- *a* + adjective beginning with a consonant + noun
- *an* + adjective beginning with a vowel + noun
- *a* + adjective beginning with a consonant sound + noun
  Example: “a European country”

Specific & Generic Use of Articles
- *a, an, the* can be used to refer to a noun that belongs to a whole class to which individual countable nouns belong.
  Example: “A penguin is a cute animal.”
  Example: “The penguin is a cute animal.”
- The omission of articles express generic meaning
  Example: “Elephants are large animals.” (pl. noun)
  Example: “Happiness is a productive emotion.” (noncount noun)

Practice Exercises

**Directions for Exercises 1-10:** Complete the sentences with the correct definite or indefinite article.

1. I’m in a hurry, so maybe I’ll place ______ order for takeout tonight.
2. She is one of ______ smartest people I know.
3. Have you ever bought ______ airplane ticket?
4. Please speak ______ little louder.
5. What is ______ name of the next station?
6. I have never seen ______ anteater.
7. Did you figure out ______ answer to question number eleven?
8. Is there ______ public telephone near here?
9. He left his country and came to live in ______ United States.
10. My little sister can’t ride ______ bike until she’s a bit taller.
Adjectives

- Adjectives modify nouns and pronouns. Most adjectives answer questions: What kind? Which one? How many?
- Adjectives answering the What kind? question are descriptive. They tell the quality, kind or condition of the nouns or pronouns they modify.
  - red convertible
  - dirty fork
  - noisy muffler
  - wild roses
  - The rain is gentle. Bob was tired.
- Adjectives answering the Which one? question narrow or restrict the meaning of a noun. Some of these are pronouns that become adjectives by function.
  - my money
  - our ideas
  - this reason
  - these apples
- Adjectives answering the How many? question are numbering words.
  - some people
  - each pet
  - five dollars
  - many goals

Adverbs

- Adverbs modify verbs, adjectives, and other adverbs. Adverbs answer the questions How? Where? When? and To what degree?
  - Modifying Verbs: They did [verb] their work quickly [adverb].
  - Modifying Adjectives: They were reasonably [adverb] happy [adjective].
- Adverbs answering the “How?” question are concerned with manner or way.
  - She ate the snails hungrily. He snored noisily.
- Adverbs that answer the “Where?” question show location.
  - They drove downtown. She walked upstairs.
- Adverbs answering the “When?” question indicate time.
  - The ship sailed yesterday. I expect an answer soon.
- Adverbs that answer the “To what degree?” question express extent.
  - She is entirely correct. He was somewhat annoyed.
- Most words ending in -ly are adverbs.
  - He worked skillfully. She answered him courteously.
- However, there are exceptions:
  - The house had a lovely [adjective] view.
  - Your goblin mask is ugly [adjective].
Practice Exercises

Directions for Exercises 1-10: Underline the correct adjective or adverb.

1. Chris is a (good, well) football player, but he didn’t play (good, well) in his last game.
2. She cooks (good, well) enough to have her own cooking show on television.
3. Tom’s (shocking, shockingly) bad behavior surprised all of us.
4. He plays both the drums and the flute (good, well).
5. Charlie returned his phone messages (prompt, promptly).
6. Because of the construction outside, it was (real, really) hard to concentrate during the lecture.
7. Tim borrowed (this, these) book from his teacher.
8. Fay and Sue (recent, recently) visited Paris.
9. After several sleepless nights, Nick had a (good, well) night’s sleep last night.
10. Katie purchased (this, those) products on the Internet.

Apostrophes

An apostrophe is a signal telling the reader that a word is either a possessive or a contraction. It is used with nouns and indefinite pronouns to show possession, to show the omission of letters and figures in contractions, and to form the plurals of letters, figures, and words referred to as words.

Forming Possessives

- The apostrophe is used to mark the possessive. The possessive tells the reader that someone or something owns or possesses the thing that comes after the possessive. Here are some common rules about how to correctly use apostrophes to form a possessive:
- A possessive shows that something is owned by someone. Use an apostrophe and –s to form the possessive of a noun, singular or plural, that does not end in –s.
  
  - man’s coat  
  - women’s suits
- Use an apostrophe alone to form the possessive of a plural noun ending in –s.
  
  - girls’ clothes  
  - the Browns’ house
- Use an apostrophe and –s or the apostrophe alone to form the possessive of singular nouns ending in –s. Use the apostrophe and –s only when you would pronounce the s.
James’ hat or (if you would pronounce the s) James’s hat

- Use an apostrophe and – s to form the possessive of certain indefinite pronouns.
  everybody’s idea one’s meat another’s poison

**Creating Contractions**

A contraction is simply two words collapsed into one. You use contractions most often in informal or personal types of writing but usually not in more formal types. When you speak, you use contractions every day. Here are some common rules about how to use apostrophes properly when creating contractions:

- Use an apostrophe to indicate that letters or figures have been omitted.
  o’clock (short for of the clock) in the ‘80s (short for 1980s)

- Use an apostrophe with pronouns only when you are making a contraction. A contraction is a combination of two words. The apostrophe in a contraction indicates where a letter has been omitted.
  it is = it’s she has = she’s you are = you’re

- If no letters have been left out, don’t use an apostrophe.
  **WRONG** The dog bit it’s tail. (not a contraction)
  **RIGHT** The dog bit its tail.
  **WRONG** Whose the leader now?
  **RIGHT** Who’s the leader now? (a contraction of who is)
  **WRONG** Its a big problem.
  **RIGHT** It’s a big problem. (a contraction of it is)

⚠️ Remember that the apostrophe marks the missing letter. Don’t make the common mistake of placing the apostrophe between the two words, such as in these cases: shouldn’t, do’nt, etc.

**Making Letters and Numerals Plural**

Finally, the apostrophe is used in one other way. Although the apostrophe is never used to make a word plural, it is used to make letters and numerals plural:

- Use an apostrophe to indicate the plural of letters, figures, and words used as words.
  Dot your i’s. five 8’s and’s
Practice Exercises

Directions for Exercises 1-5: On the line provided, change the following phrases into possessives (with an apostrophe or an apostrophe and an –s where necessary). Indicate all apostrophes clearly.

1. the chalk of the teachers: ________________________________
2. the flowers of the patients: ______________________________
3. the book of Alice: _______________________________________
4. the pulpit of the minister: ________________________________
5. the rights of the women: _________________________________

Directions for Exercises 6-10: On the line provided, write each of the following as a contraction.

6. should not: ________________
7. you are: ________________
8. I will: ________________
9. he has: ________________
10. will not: ________________

Directions for Exercises 11-15: In the following sentences, supply the missing apostrophes.

11. Wheres my briefcase?
12. The dean ignored the mothers reaction to the new dress code.
13. June says I have my fathers eyes.
14. Cant we eat at your buddys restaurant?
15. Jans trunk wouldnt open.
Verbs

Tense
There are twelve verb tenses in English. That means there are twelve different ways to describe an action, depending on when it takes place. Knowing how to use verb tense correctly can help you avoid unnecessary verb shifts in your writing. All this may seem complicated, but most verbs can be classified in terms of the simple tenses—present, past, and future—and seeing how they fall into this pattern will make them easier to understand. Below is a brief review of the twelve verb tenses.

Simple Tenses
Present: refers to a situation that occurs now
Example: I, we, you, they drive; He, she, it drives.
Past: expresses an action that was completed in the past
Example: I, we, you, he, she, it, they drove.
Future: expresses an action that will occur in the future
Example: I, we, you, he, she, it, they will drive.

Progressive Tenses
Present Progressive: describes an event in progress
Example: I am driving; He, she, it is driving; You, we, they are driving.
Past Progressive: describes an event that was in progress in the past
Example: I, he, she, it was driving; You, we, they were driving.
Future Progressive: describes an event that will take place in the future time
I, we, you, he, she, it, they will be driving.

Perfect Tenses
Present Perfect: refers to a time prior to now
Example: I, we, you, they have driven; He, she, it has driven.
Past Perfect: viewpoint on some past time
Example: I, we, you, he, she, it, they had driven.
Future Perfect: viewpoint on some future time
Example: I, we, you, he, she, it, they will have driven.

Perfect Progressive Tenses
Present Perfect Progressive: an action that began in the past that continues up to the present.
Example: I, we, you, they have been driving; He, she, it has been driving.
Past Perfect Progressive: an action taking place in the past prior to some other past
event
Example: I, we, you, he, she, it, they had been driving.
Future Perfect Progressive: Habitual action that begins in the present and will continue into the future
Example: I, we, you, he, she, it, they will have been driving.
   - Remember that irregular verbs do not follow the same pattern in their form as regular verbs.

Practice Exercises

Directions for Exercise 1: Although the main tense in the following paragraph is past, the writer correctly shifts to present tense twice. Find these two verbs in present tense. If you encounter difficulty, try reading the paragraph aloud.

1. The Iroquois Indians of the Northeast regularly burned land to increase open space for agriculture. In fact, the early settlers of Boston found so few trees that they had to row out to the islands in the harbor to obtain fuel. Just how far north this practice extended is uncertain, but the Saco River in southern Maine appears to have been the original northern boundary of the agricultural clearings. Then, pressured by European settlement, the Iroquois extended their systematic burning far northward, even into the Maritime Provinces of Canada. (abridged from Hay and Farb, The Atlantic Shore)

Directions for Exercise 2: Read the following paragraph through, and determine the main tense. Then reread it and circle the three verbs that shift incorrectly from the main tense.

2. For the past seven years, I have called myself a swimmer. Swimming, my one sport, provides a necessary outlet for my abundant energy. I have always drawn satisfaction from exertion, straining my muscles to their limits. I don’t know why pushing forward in the water, as my muscles cried out in pain, sets off a booming cheer in my head. Many times when I rounded the turn for the last lap of a race, my complaining muscles want to downshift and idle to the finish. My mind, however, presses the pedal to the floor and yells, "FASTER!" The moment that I touched the wall my muscles relax; the pain subsides. I am pleased to have passed the point of conflict. (adapted from Brendon MacLean, "Harder!")
Directions for Exercise 3: You will notice several shifts in tense in the following paragraph describing action in a fictional narrative. Find the six faulty shifts in tense.

3. In "The Use of Force" William Carlos Williams describes a struggle involving a doctor, two parents, and their young daughter. The doctor must obtain a throat culture from the girl, who was suspected of having diphtheria. This ordinarily simple task is hindered by the frightened and uncooperative patient, Mathilda Olson. Adding to the doctor's difficulties were the parents, who had to struggle with their own conflicting emotions. They want their daughter helped, but they did not trust the doctor to do the right thing. Sensitive to the parents' uncertainty, the doctor became more and more frustrated by Mathilda's resistance. Williams gives considerable attention to how each of the Olsons react, but it is clear that his main interest was in the doctor and his responses. (adapted from a student essay)

http://engnet.jiangnan.edu.cn/zhxl/xzmj/owl_purdue/g_tensecEX1.html

Recognizing Subjects and Verbs (Adapted from Wordsmith, 2007, Pamela, Arlov)
To find the subject, ask the question “Who or what ______?”
To find the subject of the verb, ask the question “Who or what______?”

- Look for the word that expresses action.
  Example: She applied for a job in a different department.
- A verb will often link the subject to the rest of the sentence.
  Example: The roller coaster looked dangerous.
- A verb may also include a “helping verb” or a “linking verb.”
  Example: They had not planned on buying a new car.
- Some verbs are compound verbs.
  Example: They had not calculated the payments before they applied for the loan.
- An infinitive form (to + go) does not act as a verb in a sentence.
  Example: Angela has begun to work from home. (“to work” cannot be the main verb. The main verb is “begun.”)
- When a verb ends in –ing, it cannot act as the web in the sentence unless a helping verb precedes it.
  Example: Brian was driving way to fast.

(Note: You can often find the subject and verb in a sentence by eliminating any prepositional phrases.)
Subject and Verb Agreement

When writing, remember that subjects and verbs have to agree. Below is a list of reminders for subject/verb agreement.

- When the subject of a sentence is composed of two or more nouns or pronouns connected by **and**, use a plural verb. (**AND** implies addition. The nouns or pronouns in the subject are “added” together, so they require a plural verb.)
- When two or more singular nouns or pronouns are connected by **or** or **nor**, use a singular verb. (**OR** and **NOR** imply only one option, so a singular verb is needed.)
- When a compound subject contains both a singular and a plural noun or pronoun joined by **or** or **nor**, the verb should agree with the part of the subject that is nearer the verb.
- Do not be misled by a phrase that comes between the subject and the verb. The verb agrees with the subject, not with a noun or pronoun in the phrase.
- The words **each**, **each one**, **either**, **neither**, **everyone**, **everybody**, **anybody**, **anyone**, **nobody**, **somebody**, **someone**, and **no one** are singular and require a singular verb.
- Nouns such as **civics**, **mathematics**, **dollars**, **measles**, and **news** require singular verbs. (The ‘s’ at the end of these words does not mean they are plural; it’s part of their spelling.) Note: the word **dollars** is a special case. When talking about an amount of money, it requires a singular verb, but when referring to the dollars themselves, a plural verb is required.
- Nouns such as **scissors**, **tweezers**, **trousers**, and **shears** require plural verbs. (There are two parts to these things.)
- In sentences beginning with **there is** or **there are**, the subject follows the verb. Since **there** is not the subject, the verb agrees with what follows.
- Collective nouns are words that imply more than one person but that are considered singular and take a singular verb, such as: **group**, **team**, **committee**, **class**, and **family**.
- Expressions such as **with**, **together with**, **including**, **accompanied by**, **in addition to**, or **as well** do not change the number of the subject. If the subject is singular, the verb is too.

[http://owl.english.purdue.edu/owl/resource/599/01/](http://owl.english.purdue.edu/owl/resource/599/01/)
Practice Exercises

Directions for Exercises 1-20: For each sentence, circle the correct verb form.

1. Ron (works, work) in a tractor factory.
2. Mail-order catalogs (makes, make) shopping easier.
3. She (has, have) a stressful job as an emergency medical technician.
4. My golden retriever puppy (grows, grow) bigger every week.
5. Drinking and driving (does, do) not mix.
6. Her estimate for the repairs (seems, seem) high.
7. When (is, are) your in-laws coming to visit?
8. The students who (lives, live) next door have loud parties.
9. Harold and Maude (expects, expect) to get married soon.
10. Your brother (budgets, budget) his salary well.
11. It (bothers, bother) me when you brag that way.
12. Each of the winners (receives, receive) $1000.
13. Why (does, do) so many divorces happen in the United States?
14. There (is, are) sandwiches in the refrigerator.
15. The billboards on this road (spoils, spoil) the view.
16. Which (is, are) your convertible?
17. One of the clocks (shows, show) the correct time.
18. Where (has, have) you stacked the sale items?
19. I prefer guests who (doesn’t, don’t) smoke.
20. Neither of these computer courses (offers, offer) what I need.

Pronouns

Pronouns take the place of or refer to nouns or other pronouns. The word or words that the pronoun refers to is called the antecedent.

Types of Pronouns
- Subject and Object
- Reflexive
- Possessive
- Indefinite
- Demonstrative
Subject and Object Forms of Personal Pronouns

<table>
<thead>
<tr>
<th>Person</th>
<th>Subject</th>
<th>Singular</th>
<th>Plural</th>
<th>Object</th>
<th>Singular</th>
<th>Plural</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st person</td>
<td>I</td>
<td>me</td>
<td>us</td>
<td>me</td>
<td>me</td>
<td>us</td>
</tr>
<tr>
<td>2nd person</td>
<td>you</td>
<td>you</td>
<td>you</td>
<td>you</td>
<td>you</td>
<td>you</td>
</tr>
<tr>
<td>3rd person</td>
<td>she/he/it</td>
<td>they</td>
<td>her/him/it</td>
<td>them</td>
<td>them</td>
<td></td>
</tr>
</tbody>
</table>

Example: *He* loves going skiing in the winter. (subject)

Reflexive Pronouns

<table>
<thead>
<tr>
<th>Person</th>
<th>Singular</th>
<th>Plural</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st person</td>
<td>myself</td>
<td>ourselves</td>
</tr>
<tr>
<td>2nd person</td>
<td>yourself</td>
<td>yourselves</td>
</tr>
<tr>
<td>3rd person</td>
<td>herself/himself/itself</td>
<td>themselves</td>
</tr>
</tbody>
</table>

Example: Jane bought *herself* a new dress.

Possessive Pronouns and Possessive Adjectives

<table>
<thead>
<tr>
<th>Person</th>
<th>Singular</th>
<th>Plural</th>
<th>Singular</th>
<th>Plural</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st person</td>
<td>my (book)</td>
<td>our (book)</td>
<td>mine</td>
<td>ours</td>
</tr>
<tr>
<td>2nd person</td>
<td>your (book)</td>
<td>your (book)</td>
<td>yours</td>
<td>yours</td>
</tr>
<tr>
<td>3rd person</td>
<td>her/his/its</td>
<td>their</td>
<td>hers/his</td>
<td>thiers</td>
</tr>
</tbody>
</table>

Possessive adjectives are possessive forms of personal pronouns that must be followed by a noun.
Example: That is *my* book.

Possessive pronouns stand alone and reference who the noun belongs to.
Example: The book is *mine*.

Indefinite Pronouns

<table>
<thead>
<tr>
<th>-body</th>
<th>some</th>
<th>any</th>
<th>No</th>
<th>every</th>
</tr>
</thead>
<tbody>
<tr>
<td>-one</td>
<td>somebody</td>
<td>anybody</td>
<td>nobody</td>
<td>everybody</td>
</tr>
<tr>
<td>-thing</td>
<td>something</td>
<td>anyone</td>
<td>no one</td>
<td>everyone</td>
</tr>
</tbody>
</table>

Indefinite pronouns are usually singular.
A pronoun that refers to indefinite pronouns should be singular.
Example: If *someone* says something nice, thank *him* or *her*. 
Demonstrative Pronouns & Adjectives

<table>
<thead>
<tr>
<th>Proximity</th>
<th>Singular</th>
<th>Plural</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near</td>
<td>this</td>
<td>these</td>
</tr>
<tr>
<td>Far</td>
<td>that</td>
<td>those</td>
</tr>
</tbody>
</table>

Demonstrative pronoun points to and identifies a noun or a pronoun.
Example: *This* has been a difficult year for Sarah.

Demonstrative pronouns may act as adjectives and modify a noun rather than stand for it.
Example: *Those plants* are poisonous.

Practice Exercises

Directions for Exercises 1-25: Using the information provided above, fill in the blank with the correct pronoun.

1. ______ apples are delicious.
2. The hikers spotted ______ dogs at the end of the trail.
3. The club members agreed to pay ______ dues on time.
4. Please go over ______ before you take the test.
5. Susanne kissed ______.
6. These are ______ books.
7. That book is ______.
8. Jeff will put ______ on display.
9. After the PTA meeting, the school board announced ______ choice for Teacher of the Year.
10. Bobby took ______ dog for a walk around the block.
11. The boys thought of ______ as daring and courageous.
12. He will put a new part on ______ if necessary.
13. Andrea took ______ things and left.
14. Many of our neighbors leave ______ doors unlocked all of the time.
15. ______ computer application will not work.
16. The football team won ______ first victory.
17. Anybody who played in the championship soccer game will get ______ own trophy.
18. ______ should do *his or her* part to protect the environment.
19. The girls thought of ______ as good athletes.
20. If anyone wants to ask me a question about the test, ______ can see me after school.
21. Many artisans sell ______ crafts for discounted prices at the fair.
22. The book lost ______ cover during the move yesterday.
23. One of the actresses forgot ______ lines during the play.
24. Those are ______ books.
25. ______ path goes in the wrong direction.

III. Vary Sentence Structure

When writing your essay, you should use different sentences structures to add complexity and variation to your writing.

- Alternate using simple sentences (sentences with one independent clause), compound sentences (sentence with multiple independent clauses but no dependent clauses), complex sentences (a sentence with one independent clause and at least one dependent clause, complex-compound sentence (sentence with multiple independent clauses and at least one dependent clause) to add variety and complexity to your writing.
- Vary sentence openings to change sentence structure and emphasis.

IV. Rhetorical Skills: Writing Strategies, Organization, Style

Prewriting & Planning

- Think about your topic and write down any ideas you have about the topic presented.
- Consider the various points of view on the topic and what supporters, and those with opposite points of view, might say and write them down.
- Organize and diagram your ideas on paper before you start to write.
- Do not go off topic.
Essay Organization

Below is an example of how an essay might look. When planning and writing your essay, be sure your essay is coherent, logical, and has plenty of supporting details to support your position.

Introduction

- An introduction typically includes an opening sentence to get the reader’s attention, background on the subject, and a thesis statement or position on the issue.

Body

- Body paragraphs each include a topic sentence that states thesis point/position you wish to develop.
- Each topic sentence should be supported with evidence, examples, explanations, and information appropriate to the thesis or position.
- When it is appropriate address counterarguments.
- At the end of each paragraph include a summary sentence.
  *Remember to use transition words within the body paragraphs and between the paragraphs, and to think about your audience when developing your essay.

Conclusion

- Begin the paragraph with a thesis/position restatement to take the reader back to the broad picture.
- Make a final closing statement.

Style

- Remember to consider your audience when writing your paper.
- Use language that is clear, concise, and is appropriate to the audience you are writing the paper for.
- Remember to use verb tense consistently and clearly.

Editing

Step 1

- Begin at the beginning of the essay and check that transitions between ideas are smooth.
- Make sure that transitions between sentences and paragraphs flow.
- Check parallel structure and clear pronoun reference.
Step 2
- Begin at the bottom of the essay.
- Look at each sentence independently and check for grammatical errors.

Step 3
- Go through the essay focusing on any errors that are common in your writing.

Revising
- Make sure each paragraph has a topic sentence.
- Make sure you have provided support for your topic sentence with examples and explanations that provide specific details that support the topic sentence.
- Make sure the final sentence of the paragraph closes the paragraph.
- Make sure your ideas are in logical order.
- Use transition words effectively when moving from one thought to the next.

Practice Exercises Answer Key

Commas
1. While we were watching for Mom, we started our homework. (Introductory adverb clause)
2. Although she is wearing red, blue is her favorite color. (Introductory adverb clause)

Reminder: Many dependent clauses will begin with a subordinate conjunction such as the following: after, although, as if, because, before, if, since, unless, when, while, etc. If the dependent clause follows the independent clause, then a comma is not needed between the two clauses.
3. The new cars arrived yesterday, and they will be delivered tomorrow.
4. Most of the lecture was interesting, but some students were not impressed.
5. Mr. Jensen wanted us to bring apples, milk, and bread. (A list separated by commas)
6. Rehearsals are held before school, during recess, and at night. (A list of phrases separated by commas)
7. The supervisor wanted to know who had broken into the store, why they had done so, and what had been taken. (A list of clauses separated by commas)
8. Incidentally, I was not late this morning. (Introductory word)
9. Hoping she had enough money, she approached the register. (Introductory participial phrase)
10. To arrive on time, we must leave here by six. (Introductory infinitive phrase)
11. In light of our financial condition, building a house is not wise. (Introductory prepositional phrase)
12. The quality of the material, however, was beyond question.
13. Mr. Reyerson is, I think, a very successful salesman.
14. Jamie, please clean your room.
15. Everyone envied her dark green car. (No comma because the car is not green AND dark but dark green)
16. His arrogant, condescending manner annoyed us. (His manner is arrogant AND condescending—use a comma)
17. My father, who was born in southern Utah, is 83 years old. (The adjective clause “who was born in southern Utah” is not necessary to identify “my father” because the writer only has one father—use commas)
18. Officer Jones, chasing after the thief, grabbed the gun. (Participial phrase “chasing after the thief” is not necessary to identify Officer Jones—use commas)
19. Lacrosse, a sport played hundreds of years ago, is still popular. (Unnecessary appositive “a sport played hundreds of years ago” that explains the noun—use commas)
20. Nick Van Exel, the player from the Lakers, looks like a camel.
21. The award was given to Jan, Carol’s daughter.
22. American athletes Karl Malone and Michael Jordan are well-known to everyone. (Appositive “Karl Malone and Michael Jordan” are necessary to show which American Athletes are being spoken of—no commas)
23. Ever since, our supplies have been stored in a warehouse.
24. Before the rainstorm, everything was dry; after, everything looked bright and green.
25. We moved here from Ann Arbor, Michigan, three months ago. (Use a comma after the city and after the state.)
26. Mrs. Bitters, the next meeting will be on Tuesday, August 13, 2001, in Seattle. (Use a comma after every day, date, and year.)
27. “Scientific research,” she said, “has found an unhealthy diet increases your risk of cancer.” (Use a comma after a quotation that continues and after you introduce the speaker.) Reminder: Closing commas and periods always go inside quotation marks.

**Semicolons**
1. I can’t eat strawberries; they give me a rash.
2. The soccer game begins at 2 o’clock; please be on time.
3. Karen and Tom went to Clancy’s for dinner; they both enjoy eating seafood.
4. David left work early; he had a dentist appointment at 3:00 P.M.
5. Kate always wears earmuffs during the winter; her ears are extra sensitive to the cold.
6. I can build a tree house with scrap lumber; however, I must buy a rope ladder.
7. Ranchers put up barbed wire fences; otherwise, cattle will roam to other ranches.
8. Exercising daily takes hard work; nevertheless, millions of Americans do it.
9. The runner was shaking from fatigue; **however**, she finished the marathon.
10. Walking home takes an hour; **furthermore**, there is no sidewalk.
11. I am currently taking Spanish, which I enjoy; math, which I find difficult; and psychology, which is my favorite subject.
12. We visited Lima, Peru; Rio Dijonero, Brazil; and Salt Lake City, Utah.
13. The trio’s birthdays are November 17, 1971; June 9, 1975; and October 27, 1974.
14. Her favorite players are Coby Bryant, a point guard; Greg Oden, a center; and Brandon Roy, a forward.
15. The teenager attempted to purchase beer from the undercover officer on Friday, July 4th; Saturday, July 5th; and Sunday, July 6th.

**Colon**
1. Road construction in Portland has hindered travel around town: parts of Broadway, Fifth, and Sixth are closed during the construction.
2. Brian was never out of his family’s reach: he carried is cell phone wherever he went.
3. Mike could not remember that the accident had totaled his car and put him in the hospital, but he was sure of one thing: he was lucky to be alive.
4. Julie went to the store for some groceries: milk, coffee, cereal, and fruit.
5. I know the perfect job for her: a politician.
6. In his Gettysburg Address, Abraham Lincoln urges Americans to rededicate themselves to the unfinished work of the deceased soldiers: "It is for us the living rather to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us — that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion — that we here highly resolve that these dead shall not have died in vain, that this nation under God shall have a new birth of freedom, and that government of the people, by the people, for the people shall not perish from the earth."

**Parentheses**
1. The movie (which is rated-R) has violence and adult language.
2. Her husband says that there is no better way to spend a Sunday afternoon than eating spaghetti (his favorite food) and watching a movie.
3. She loved the color (gray) of her new car.

**Dash**
1. It was his humor--not his good looks--that made him so popular.
2. There are two reasons the couple are having problems--money and communication.
3. She had so many errands to do—go to the store, stop by the bank, and pick-up the kids—that she did not know where to begin.
4. The three sisters—Ally, Jessica, and Kirstin—fought all the time.
5. There are three types of fruit—oranges, papaya, and bananas—that I like to eat all the time.

**Quotations**
1. She asked, "When will you be leaving for the airport?" I answered, "At 6:30 tomorrow morning."
2. Apple just introduced the new version of the "iphone."

**Articles**
1. I'm in a hurry, so maybe I'll place an order for takeout tonight.
2. She is one of the smartest people I know.
3. Have you ever bought an airplane ticket?
4. Please speak a little louder.
5. What is the name of the next station?
6. I have never seen an anteater.
7. Did you figure out the answer to question number eleven?
8. Is there a public telephone near here?
9. He left his country and came to live in the United States.
10. My little sister can't ride a bike until she's a bit taller.

**Adjectives and Adverbs**
1. Chris is a (good, well) football player, but he didn't play (good, well) in his last game.
2. She cooks (good, well) enough to have her own cooking show on television.
3. Tom's (shocking, shockingly) bad behavior surprised all of us.
4. He plays both the drums and the flute (good, well).
5. Charlie returned his phone messages (prompt, promptly).
6. Because of the construction outside, it was (real, really) hard to concentrate during the lecture.
7. Tim borrowed (this, these) book from his teacher.
8. Fay and Sue (recent, recently) visited Paris.
9. After several sleepless nights, Nick had a (good, well) night's sleep last night.
10. Katie purchased (this, those) products on the Internet.
**Apostrophes**

1. the chalk of the teachers: **the teachers’ chalk**
2. the flowers of the patients: **the patients’ flowers**
3. the book of Alice: **Alice’s book**
4. the pulpit of the minister: **the minister’s pulpit**
5. the rights of the women: **women’s rights**
6. you are: **you’re**
7. I will: **I’ll**
8. he has: **he’s**
9. will not: **won’t**
10. Where’s my briefcase?
11. The dean ignored the **mother’s** reaction to the new dress code.
12. June says I have my **father’s** eyes.
13. Can’t we eat at your **buddy’s** restaurant?
14. **Jan’s** trunk **wouldn’t** open.

**Tense**

1. In this first paragraph, the two verbs in present tense--both appropriate for the situation--are indicated in **bold**.

The Iroquois Indians of the Northeast regularly burned land to increase open space for agriculture. In fact, the early settlers of Boston found so few trees that they had to row out to the islands in the harbor to obtain fuel. Just how far north this practice extended is uncertain, but the Saco River in southern Maine **appears** to have been the original northern boundary of the agricultural clearings. Then, pressured by European settlement, the Iroquois extended their systematic burning far northward, even into the Maritime Provinces of Canada. (abridged from Hay and Farb, *The Atlantic Shore*)

2. The main tense in this next paragraph is present. Incorrect shifts to past tense are indicated in **bold**.

For the past seven years, I have called myself a swimmer. Swimming, my one sport, provides a necessary outlet for my abundant energy. I have always drawn satisfaction from exertion, straining my muscles to their limits. I don’t know why pushing forward in the water, as my muscles **cried out** in pain, sets off a booming cheer in my head. Many times when I **rounded** the turn for the last lap of a race, my complaining muscles want to downshift and idle to the finish. My mind, however, presses the pedal to the floor and yells, "FASTER!" The moment that I **touched** the wall my muscles relax; the pain subsides. I am pleased to have passed the point of conflict. (adapted from Brendon MacLean, "Harder!")
3. Since the following paragraph describes action in a fictional narrative, the main tense should be present. The six incorrect shifts to past tense are in bold.
In "The Use of Force" William Carlos Williams describes a struggle involving a doctor, two parents, and their young daughter. The doctor must obtain a throat culture from the girl, who was suspected of having diphtheria. This ordinarily simple task is hindered by the frightened and uncooperative patient, Mathilda Olson. Adding to the doctor's difficulties were the parents, who had to struggle with their own conflicting emotions. They want their daughter helped, but they did not trust the doctor to do the right thing. Sensitive to the parents' uncertainty, the doctor became more and more frustrated by Mathilda's resistance. Williams gives considerable attention to how each of the Olsons react, but it is clear that his main interest was in the doctor and his responses. (adapted from a student essay)

Subject and Verb Agreement
1. Ron (works, work) in a tractor factory.
2. Mail-order catalogs (makes, make) shopping easier.
3. She (has, have) a stressful job as an emergency medical technician.
4. My golden retriever puppy (grows, grow) bigger every week.
5. Drinking and driving (does, do) not mix.
6. Her estimate for the repairs (seems, seem) high.
7. When (is, are) your in-laws coming to visit?
8. The students who (lives, live) next door have loud parties.
9. Harold and Maude (expects, expect) to get married soon.
10. Your brother (budgets, budget) his salary well.
11. It (bothers, bother) me when you brag that way.
12. Each of the winners (receives, receive) $1000.
13. Why (does, do) so many divorces happen in the United States?
14. There (is, are) sandwiches in the refrigerator.
15. The billboards on this road (spoils, spoil) the view.
16. Which (is, are) your convertible?
17. One of the clocks (shows, show) the correct time.
18. Where (has, have) you stacked the sale items?
19. I prefer guests who (doesn’t, don’t) smoke.
20. Neither of these computer courses (offers, offer) what I need.

Pronouns
1. These apples are delicious.
2. The hikers spotted their dogs at the end of the trail.
3. The club members agreed to pay their dues on time.
4. Please go over these before you take the test.
5. Susanne kissed him.
6. These are our books.
7. That book is yours.
8. Jeff will put those on display.
9. After the PTA meeting, the school board announced its choice for Teacher of the Year.
10. Bobby took his dog for a walk around the block.
11. The boys thought of themselves as daring and courageous.
12. He will put a new part on that if necessary.
13. Andrea took her things and left.
14. Many of our neighbors leave their doors unlocked all of the time.
15. That computer application will not work.
16. The football team won its first victory.
17. Anybody who played in the championship soccer game will get his or her own trophy.
18. Everyone should do his or her part to protect the environment.
19. The girls thought of themselves as good athletes.
20. If anyone wants to ask me a question about the test, he or she can see me after school.
21. Many artisans sell their crafts for discounted prices at the fair.
22. The book lost its cover during the move yesterday.
23. One of the actresses forgot her lines during the play.
24. Those are our books.
25. This path goes in the wrong direction.

Texts and websites consulted for information include:

OWL at Purdue University:  http://owl.english.purdue.edu
Wordsmith, 2007, Pamela Arlov

Information, examples, and some practice exercises for this review packet were barrowed from OWL at Purdue University website and from handouts available through the Student Success Center at Portland Community College, Portland, OR.
Online Grammar & Writing Resources

- Capital Community College-Sentence Sense: http://webster.commnet.edu/sensen/ and http://grammar.ccc.commnet.edu/grammar/

  This website offers online instruction for a wide range of grammatical topics.

- Jack Lynch’s Grammar & Style Notes: http://andromeda.rutgers.edu/~jlynch/Writing/
  This website is an online guide to grammar and style. Information is easily accessed through links on the table of contents.

- McGraw-Hill Online Learning Center:
  http://www.mhhe.com/socscience/english/langan/langan_7_es/
  The Student Center offers English skill practice in 52 chapters of instruction in writing and grammar. Each chapter includes objectives, chapter outline, self-quizzes, glossary, online resources, and visuals.

- Purdue Online Writing Lab:
  http://owl.english.purdue.edu/handouts/grammar/#grammar
  http://owl.english.purdue.edu/workshops/pp/index.html
  A comprehensive website that offers instruction and online practice exercises for a wide variety of grammatical exercises. A number of writing specific power point presentations are also available for viewing.

- Townsend Press Learning Center: http://www.townsendpress.net/home.php
  This website offers online exercises to supplement the grammar textbook English Essentials. Textbook is not needed to do these lessons.

- Word Surfing: http://www.wordsurfing.co.uk/19.html
Math Review

The Math Review is divided into two parts:

I. The first part is a general overview of the math classes, their sequence, basic content, and short quizzes to see if you are prepared to take a particular class. – page 50

II. The second part is a refresher of some basic topics for those who know how but lost their fluency over the years. – page 88

All information in Part I is taken from flyers created by the PCC Math SAC.
Portland Community College
SEQUENCE OF MATH COURSES

MATH 10 - Fundamentals of Arithmetic
2cr/2hr

MATH 20 - Basic Math
4cr/4hr

MATH 30 - Business Math
4cr/4hr

MATH 60 – Introductory Algebra
1st Term

MATH 65 – Introductory Algebra
2nd Term
4cr/5hr (each)

OR

MATH 61 – 62 - 63
Intro to Algebra
Part I, Part II, & Part III
(Optional sequence)
3cr/4hr (each)

MATH 70 - Introduction to Intermediate Algebra
(Optional course – consult a Math advisor)
4cr/5hr

MATH 95
Intermediate Algebra
4cr/5hr

OR

MATH 91 -92
Intermediate Algebra
Part I & Part II
(Optional sequence)
3cr/4hr each

MATH 105
Exploration in Mathematics
4cr/4hr

MATH 111B
College Algebra for Business, Life Science, & Social Science Majors
5cr/5hr

MATH 111C
College Algebra for Engineering, Math & Science Majors
5cr/5hr

MATH 112
Elementary Functions (Trigonometry)
5cr/5hr

MATH 231
Elements of Discrete Math I
4cr/4hr

MATH 232
Elements of Discrete Math II
4cr/4hr

MATH 241
Calculus for Business, Life Science & Social Science Majors
4cr/4hr

MATH 243
Statistics I
4cr/4hr

MATH 244
Statistics II
4cr/4hr

MATH 245
Statistics III
4cr/4hr

MATH 251 (Lec & Lab)
Calculus I
4cr/8hr

MATH 252
then
MATH 253
Calculus II & Calculus III
5cr/5hr (each)

MATH 254
Vector Calculus
5cr/5hr

MATH 256
Differential Equations
5cr/5hr

MATH 257
Advanced Calculus
5cr/5hr

MATH 258
Partial Differential Equations
5cr/5hr

MATH 261
Applied Linear Algebra
5cr/5hr

MATH 262
Linear Algebra
5cr/5hr

MATH 263
Linear Algebra for Engineers
5cr/5hr

MATH 264
Linear Algebra for Scientists
5cr/5hr

MATH 265
Linear Algebra for Mathematicians
5cr/5hr

MATH 266
Linear Algebra for Engineers & Scientists
5cr/5hr

EFFECTIVE PREREQUISITE SEQUENCE
Math Review – Part I

On the previous page you found a flow chart of the math sequence. The placement test will determine where you enter the sequence. Your educational goal will determine where you will exit the sequence. Please see an advisor for that.

In the following pages the topics are listed that are covered in each class.

**TOPICS**

covered in PCC Math Classes

To be successful studying the topics covered in these courses, students should be appropriately prepared by:

1. Taking the prerequisite math course within the last three years with a passing grade of A or B, or within the last one year with a passing grade of C, or
2. Placing into the course by taking the COMPASS placement test.

**MTH 20 – Basic Math**

*Fractions, Decimals, Integers* ⇒ addition, subtraction, multiplication, division, Order of operations

*Ratio and Proportion*

*Percent* ⇒ percents ⇆ decimals ⇆ fractions

*Measurements* ⇒ Metric system ⇆ American system

*Geometry*

*Statistics*

Place value, rounding, inequalities, exponents, power of ten
Prime numbers, multiples, prime factorization, least common multiples
MTH 60 – Introductory Algebra I

I. Integer arithmetic
   a. The four basic operations of addition, subtraction, multiplication, and division
   b. Absolute value, exponents, order of operations

II. One variable linear equations and inequalities

III. Application (i.e. word/story) problems with formulas

IV. Graphing lines
   a. Finding and interpreting slope
   b. Finding and interpreting intercepts
   c. Interpret relationships between variables
   d. Modeling with linear equations

MTH 65 – Introductory Algebra II

1. Systems of linear equations in two variables
   a. Graphing method
   b. Substitution method
   c. Addition method
   d. Applications

2. Working with algebraic expressions
   a. Add, subtract, multiply, and divide by a monomial
   b. Factoring polynomials

3. Solving quadratic equations
   a. Square Root Property (includes – simplify and approximate numeric square roots)
   b. Factoring Property
   c. Quadratic Formula
   d. Graphing (includes – interpret vertex, axis of symmetry and vertical/horizontal intercepts)
   e. Applications

4. Relations and functions
   a. Function notation
   b. Evaluate
MTH 70 – Review of Introductory Algebra

1. Solving equations
   A. Linear equations
   B. Quadratic equations
   C. Rational equations
   D. Radical equations
2. Graphing
   A. Linear functions
   B. Quadratic functions
3. Simplifying expressions
   A. Polynomial expressions
   B. Rational expressions
4. Function concepts
   A. Domain
   B. Range
   C. Function notation
   D. Graph reading

MTH 95 – Intermediate Algebra

1. Applications and Modeling
   A. Linear functions
   B. Quadratic functions
   C. Exponential functions
2. Graphing
   A. Linear functions
   B. Quadratic functions
   C. Exponential functions
3. Solve equations and inequalities
   A. Symbolically
   B. Numerically
   C. Graphically
4. Function concepts
   A. Domain
   B. Range
   C. Inverses
   D. Compositions
   E. Transformations
MTH 111 – College Algebra (MTH 111B or MTH 111C)

1. Graphing and solving equations and applications involving:
   A. Polynomial functions
   B. Rational functions
   C. Exponential functions
   D. Logarithmic functions

2. Functions Operations
   A. Inverses of functions
   B. Compositions of functions
   C. Transformations of functions

MTH 112 – Elementary Functions (Trigonometry)

1. Right triangle trigonometry
2. Law of Sines and Law of Cosines and their applications
3. Solutions to trigonometric equations
4. Applications
   A. Vectors
   B. Parametric equations
   C. Polar coordinates and graphs
   D. Complex numbers

MTH 211 – Foundations of Elementary Math I

1. Topics for Math 211:
   A. Problem solving
   B. Set Theory – union, intersection, complement, Venn Diagrams
   C. Historic Numeration Systems
   D. Whole Number Operations – properties, algorithms, models, non-decimal bases.
   E. Number Theory – divisibility, primes, GCD, LCM, modular arithmetic.
2. **Topics for Math 212:**
   A. Fractions, Decimals, Percents – operations, models, algorithms, problem solving.
   B. Real Number System
   C. Probability – modeling, multistage experiments, methods of counting.
   D. Introductory Statistics – data, graphs, averages

3. **Topics for Math 213:**
   A. Introductory Geometry – curves, angles, congruence, constructions, 3-dimensional.
   B. Transformational Geometry – translation, rotation, reflection, tessellations
   C. Measurement Concepts – length, area, volume
   D. Metric System – meter, gram, liter, Celsius

**MTH 241 – Calculus for Business, Life Science, and Social Science**

1. Evaluating limits of functions

2. Continuity of functions
   A. Continuity at a point
   B. Intervals of continuity
   C. Removable and essential discontinuities
   D. Viewing practical situations in terms of a continuous function

3. Differentiation
   A. Definition of a derivative function
   B. Rules for finding the derivatives of algebraic, exponential, and logarithmic functions
   C. Implicit differentiation
   D. Logarithmic differentiation
   E. Higher order derivatives

4. Applications of differentiation
   A. Graphing
   B. Extrema problems
   C. Business applications
5. Integration
   A. Definition of the definite integral and the indefinite integral
   B. Techniques for evaluating the indefinite integral and the definite integral
   C. Applications of integration in problems related to business
   D. Approximate integration
   E. Solving differential equations

MTH 243 – Statistics I

1. Describe Data
   A. Construct and interpret graphical displays
   B. Calculate and interpret numerical summaries
2. Produce Data
   A. Experiments and observational studies
   B. Randomization
   C. Sampling design
3. Probability
   A. Randomness
   B. Probability models
   C. Random variables
4. Sampling Distributions
   A. Counts and proportions
   B. Sample means
5. Estimation
   A. Confidence interval for a population mean
   B. Sample size

MTH 251 – Calculus I

1. Limits
2. Differentiation
3. Applications to Differentiation
4. Numerical Integration
MTH 252 – Calculus II

Skills:
1. Use of summation signs
2. Limit of summations as \( n \to \infty \)
3. Use of Reimann sums to find area
4. Interpret Reimann sum as definite integral
5. Fundamental Theorem of Calculus
6. Power rule for integration
7. Constant of integration
8. The Trapezoid rule
9. Simpson’s rule

Applications:
1. The area between two graphs using either \( dx \) or \( dy \)
2. Volume using Disks and Washers, shells or slicing
3. Area of surface of revolution
4. Arc Length
5. Work required
6. Water pressure

MTH 253 – Calculus III

1. Powerful integrating techniques including
   A. By parts
   B. Special Powers
   C. Trigonometric substitutions
   D. Quadratics
   E. Partial fractions
   F. Trapezoid rule
   G. Simpson’s rule

2. Improper Integrals
   A. How to deal with “holes”, vertical tangents and disjoint functions.

3. Limits of Indeterminate Forms
   A. L’Hopital
   B. Reciprocal rule
   C. Logarithmic Process
4. Sequences and series
   A. Limits
   B. Convergence and divergence
   C. Power series
5. Polar coordinates
6. Parametric equations

ARE YOU PREPARED?

✓ The mini quizzes on the following pages are meant to serve only as an indicator of a few of the math skills that you are expected to know at the beginning of each course. Do not use these problems as a study guide thinking that they will adequately prepare you for the course.

✓ These example problems are merely representative of some of the most important concepts that are taught in the prerequisite courses.

✓ The courses will offer little or no time for any type of review; they assume that you are prepared to do the work the first day of class.

Below is a sample of some skills you should have BEFORE entering

MTH 20 – Basic Math

You MAY NOT use a calculator.

1. Without using a calculator, can you complete these problems in 45 seconds?

   \[
   \begin{align*}
   6 \times 4 & \quad 9 \times 6 & \quad 7 \times 8 & \quad 9 \times 9 & \quad 0 \times 6 \\
   6 \times 9 & \quad 8 \times 10 & \quad 9 \times 4 & \quad 6 \times 7 & \quad 7 \times 2 \\
   9 \times 0 & \quad 6 \times 2 & \quad 4 \times 7 & \quad 8 \times 9 & \quad 9 \times 7
   \end{align*}
   \]
6 x 5  8 x 9  8 x 4  3 x 6  8 x 8  
12 ÷ 4  56 ÷ 8  72 ÷ 9  40 ÷ 5  36 ÷ 6

NOTE: If you miss more than 5 problems, then you should consider taking the previous math course – ABE 0750 or ALC 60, 61, 62, 63.

2. Without using a calculator, can you get at least 8 correct answers on the following problems?
   a) 20 x 30  b) 25 + 4 + 125  c) 872 - 431
   d) 4984 ÷ 8  e) 68 x 34  f) 17575 ÷ 25
   g) 305 x 27  h) 5843 - 2338  i) 4590 ÷ 15
   j) 45 + 2,341 + 8 + 124

3. Without using a calculator, can you get at least 4 correct answers on the following problems?
   a) Find the change from a $20 bill after purchasing 2 records at $6 each, and 1 pair of earrings that cost $3.
   b) A computer screen consists of small rectangular dots called pixels. How many pixels are there on a screen that has 600 rows with 800 pixels in each row?
   c) Before going back to college, David buys 4 shirts at $59 each and 6 pairs of pants at $78 each. What is the total cost of the purchase?
   d) Portland community college is constructing new dorms. Each dorm room has a small kitchen. If someone buys 85 microwave ovens at $90 each, what is the total cost of the purchase?
   e) Hershey Chocolate USA makes small, fun-size chocolate bars. How many 20-bar packages can be filled with 8,110 bars? How many bars will be left over?
**Answers**

**QUESTION 2:**

a) 600  

b) 154  

c) 441  

d) 623  

e) 2,312  

f) 703  

g) 8,235  

h) 3,505  

i) 306  

j) 2,518

**QUESTION 3:**

a) $5  

b) 480,000 PIXELS  

c) $704  

d) $7,650  

e) 405 Packages with 10 bars left over

How many of these problems can you miss and still succeed in MTH 20?

Ideally, NONE.

These problems are just a sample of the larger number of skills that you should be familiar with **BEFORE** taking this course.

If some of these ideas are not familiar to you, you should consider enrolling in one of the previous math courses (ABE 0750 or ALC 60, 61, 62, or 63).

Below is a sample of some skills you should have **BEFORE** entering

**MTH 60 – Introductory Algebra I**

You **MAY NOT** use a calculator.

1. **Without using a calculator,** can you get at least 16 correct answers on the following problems?

   a) Round 6.8449 to the nearest hundredth.  

   b) Round 7.995 to the nearest tenth.
c) Round 37,328 to the nearest hundred.
d) Change 0.625 to a fraction

e) Write 70% as a fraction and reduce to the lowest terms.
f) Change $\frac{2}{5}$ to a decimal.

g) Multiply: $\frac{9}{16} \times \frac{2}{3}
h) \text{Divide: } \frac{2}{3} \div 10$

i) Find the average of $7, 12.5, 8, 10, \frac{1}{4}$
j) Perform the indicated operations.
$7 - 3 \cdot 2 + 10 \div 5$

k) Subtract: $8.3 - 0.973$
l) Perform the indicated operations.
$18 \div 2(3) + 2^2 - 5$

m) List these numbers from smallest to largest:
$\frac{5}{9}, \frac{7}{12}, 0.555, 0.583$

n) Solve the proportion:
$\frac{2.5}{4} = \frac{1.1}{x}$

o) How many inches equal 2 yd?
p) Change 72 mg to grams.

q) If 1 km is approximately 0.6 miles, how many miles in 18 km?

r) Find the area of a circle whose diameter is 6 cm.

s) Find the perimeter of this figure:
\[\begin{array}{c}
16 \text{ m}\\
10 \text{ m}\\
8 \text{ m}
\end{array}\]
t) Find the volume of this figure:
\[\text{Volume of a rectangular prism: } \text{length} \times \text{width} \times \text{height} = 18 \text{ in} \times 25 \text{ m} \times 5 \text{ in} \]
2. Without using a calculator, can you get at least 4 correct answers on the following problems?
   a) A family’s monthly income is $1,200. It is spent as follows: 20% on food, 35% on rent, 17% on utilities, 8% on automobile, and the rest on miscellaneous expense. What dollar amount is spent on miscellaneous expenses?
   b) A TV is priced to sell at $585. What is the sale price if the sale sign says “1/3 off”?
   c) A machinist needs a bar that is \(\frac{3}{8}\) in. thick. If she cuts off \(\frac{3}{32}\) in. thick, how thick is the bar?
   d) A teacher assigns problems 96 to 128 that are multiples of 8. Which problems should the students do?
   e) Find the unit price if the total cost of a 5-lb. steak is $21.

**Answers**

**Question 1:**

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
<th>e)</th>
<th>f)</th>
<th>g)</th>
<th>h)</th>
<th>i)</th>
<th>j)</th>
<th>k)</th>
<th>l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.84</td>
<td>8.0</td>
<td>37,300</td>
<td>(\frac{5}{8})</td>
<td>(\frac{7}{10})</td>
<td>0.4</td>
<td>(\frac{3}{2})</td>
<td>(\frac{1}{6})</td>
<td>(9\frac{1}{2})</td>
<td>3</td>
<td>7.327</td>
<td>26</td>
</tr>
<tr>
<td>m)</td>
<td>n)</td>
<td>o)</td>
<td>p)</td>
<td>q)</td>
<td>r)</td>
<td>s)</td>
<td>t)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.555, (\frac{5}{9}), 0.583, (\frac{7}{12})</td>
<td>1.76</td>
<td>72 inches</td>
<td>0.072 g</td>
<td>10.8 mi</td>
<td>28.26 cm(^2)</td>
<td>44 m</td>
<td>2,250 in(^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Question 2:**

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
<th>e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$240</td>
<td>$390</td>
<td>(\frac{15}{32}) in. thick</td>
<td>96, 104, 112, 120, 128</td>
<td>$4.20 per lb</td>
</tr>
</tbody>
</table>

How many of these problems can you miss and still succeed in MTH 60?

Ideally, NONE.
These problems are just a sample of the larger number of skills that you should be familiar with **BEFORE** taking this course.

If some of these ideas are not familiar to you, you should enroll in the previous course (MTH 20 or ALC 60, 61, 62, or 63)

Below is a sample of some skills you should have **BEFORE** entering

**MTH 65 – Introductory Algebra II**

You **MAY NOT** use a calculator.

a) Perform the indicated operations:
   \[ 18 ÷ 2(-3) - (-2)^3 - 5 \]

c) Simplify:
   \[ (12x^2 - 4x + 1) - 3(2x^2 - 5x + 3) \]

e) Solve for x:
   \[ \frac{x}{2} - \frac{1}{10} = \frac{x}{5} + \frac{1}{2} \]

f) Solve for W:
   \[ P = 2L + 2W \]

i) Given two points on a line, find the slope and indicate whether the line rises, falls, is horizontal, or is vertical.
   
   and

   \[ \begin{align*}
   & \text{Given two points on a line, find the slope and indicate whether the line rises, falls, is horizontal, or is vertical.} \\
   & \text{and}
   
   & \text{k) Given the slope, } -2, \text{ and a point passing through } (1,4), \text{ write an equation in the point-slope form and slope-intercept form.}
   \end{align*} \]

b) Evaluate \( 7x - x^2 \), when \( x = -2 \)

d) Solve for x:
   \[ 5(x - 2) = 3 - 6(x - 7) \]

f) Solve for x and graph on a number line.
   \[ 2 - 6x \geq 2(5 - x) \]

h) Find the slope and the y-intercept of the line when \( 2x - y = 6 \)

j) Write an equation for the following graph.

k) Given the slope, \(-2\), and a point passing through \((1,4)\), write an equation in the point-slope form and slope-intercept form.

l) Graph the inequality on a rectangular coordinate system.
   \[ y < \frac{4}{3}x - 1 \]
This is a graph of Frank’s body temperature from 8 a.m. to 3 p.m. Let x represent the number of hours after 8 a.m. and y equal Frank’s temperature (in °F) at time x.

m) What is the y-intercept? What does this mean about Frank’s temperature at 8 a.m.?

n) During which period of time is Frank’s temperature decreasing?

o) Estimate Frank’s minimum temperature during the time period shown. How many hours after 8 a.m. does this occur? At what time does this occur?

p) How many grams of an alloy that is 80% gold should be melted with 40 grams of an alloy that is 50% gold to produce an alloy that is 70% gold?

q) Vikki has $200 to spend on clothing. She buys a skirt for $68. She would like to buy some sweaters that sell for $15.50 each. How many sweaters can she buy and stay within her budget?

r) The pressure of water on an object below the surface is proportional to its distance below the surface. If a submarine experiences a pressure of 25 pounds per square inch 60 feet below the surface, how much pressure will it experience 330 feet below the surface?

Answers

a) -24  b) -18  c) 6x² + 11x - 8  d) x = 5  e) x = 2  f) x ≤ -2

g) \[ w = \frac{p - 2L}{2} \]  h) Slope = 2, y-intercept = (0, -6)  i) Slope = 3, rises

j) x = -3  k) Point-slope form: y - 4 = -2(x + 1) and slope-intercept form: y = -2x + 2

l) \[ y < \frac{4}{3}x - 1 \]

m) The y-intercept is (0, 101). At 8:00 a.m. Frank’s body temperature is at 101 °F.
n) Frank’s temperature is decreasing during the time from 8:00 a.m. to 11:00 a.m.

o) Frank’s minimum temperature is \( \approx 98.6 \) °F. This occurs about 3 hours afterwards and the time would be 11:00 a.m.

p) Eighty grams of an alloy that is 80% gold should be melted with 40 grams of an alloy that is 50% gold to produce an alloy that is 70% gold.

q) Vikki can buy at most eight sweaters.

r) A submarine will experience a pressure of 137.5 pounds per square inch 330 feet below the surface.

How many of these problems can you miss and still succeed in MTH 65?

Ideally, NONE.

These problems are just a sample of the larger number of skills which you should be familiar with BEFORE taking this course.

If some of these ideas are not familiar to you, you should consider enrolling in the previous course (MTH 60 or ALC 60, 61, 62, or 63).

Below is a sample of some skills you should have BEFORE entering

**MTH 70 – Review of Introductory Algebra**

Part I

Work with positive and negative real numbers, fractions, and the order of operations.

a) \( 100 \div 4 \times 5 \)

b) \( \frac{(-3)(-4) - 3^3}{-4 + 6} \)

c) \( \frac{2}{3} \div \left( \frac{1}{3} + \frac{3}{8} \right) \)
Part II

1. Simplify expressions:
   a) \[3(2x^2 - 3xy + y) - (y - x^2 + 2xy)\]
   b) \[12 - 2(x - 2)\]
   c) \[\left(\frac{27x^2y^5}{9x^6y^2}\right)^3\]

2. Factor:
   a) \[x^2 - 5x - 14\]
   b) \[6a^2b^3 - 3a^2b\]

3. Solve for \(x\):
   a) \[3x - 4 + 4 = 5(x - 4) - 4\]
   b) \[3x - 5y + 6 = 0\]
   c) \[x^2 - 5x - 14 = 0\]

4. Evaluate expressions:
   If \(x = -3\), evaluate \(-x^2 - 2x - 1\)

5. Graph by HAND and on your GRAPHING CALCULATOR*
   a) \[4x + 3y = -12\]
   b) \[y = 6x^2 + 90x - 600\]

6. Find the equation of the line passing through 2 given points:
   \((2, -1)\) \((-1, -7)\)

7. Solve a first-degree inequality in one variable:
   Given: \(8 - 5x \geq 3x + 9\), solve for \(x\)

8. Given \(f(x) = -3x + 2\)
   a) Evaluate \(f(-2)\)
   b) Solve for \(x\) if \(f(x) = -2\)

Answers

Part I
   a) 125  
   b) \(\frac{3}{2}\)  
   c) \(\frac{16}{17}\)
Part II

1. a) $7x^2 - 11xy + 2y$ b) $16 - 2x$ c) $\frac{27y^9}{x^{34}}$

2. a) $(x - 7)(x + 2)$ b) $3a^2b(2b^2 - 1)$

3. a) $x = 5$ b) $x = \frac{5y - 6}{3}$ c) $x = 7, x = -2$

4. $-4$

5. a) 

6. $y = 2x - 5$

7. $x \leq \frac{1}{8}$ or $-\frac{1}{8} \geq x$

8. a) $f(-2) = 8$ b) $x = \frac{4}{3}$

*Students with no graphing calculator experience should enroll concurrently in MATH 93.

MATH 70 IS AN OPTIONAL COURSE
CONSULT A MATH ADVISOR

How many of these problems can you miss and still succeed in MTH 70?
  a) If you missed any of the problems in Part I you should consider enrolling in MTH 60.
  b) If you missed several of the problems in Part II, MTH 70 is the course for you. These topics will be reviewed in MTH 70.
  c) If you missed none of the problems, enroll in MTH 95.
Below is a sample of some skills you should have **BEFORE** entering

**MTH 95 – Intermediate Algebra**

You **MAY NOT** use a calculator, except where indicated.

1. Work with positive and negative real numbers, and the order of operations.
   Simplify 
   \[ -5 + (-4)(-3) - 3^2 \]

2. Simplify expressions:
   a) \[ 3(2x^2 - 3xy + y) - (y - x^2 + 2xy) \]
   b) \[ \frac{12a^2b^2}{8a^2b^7} \]

3. Expand and collect like terms:
   a) \((3x - 5)(6x + 7)\)
   b) \((2x - 3)^2\)

4. Factor:
   a) \(x^2 - 5x - 14\)
   b) \(6a^2b^3 - 3a^2b\)

5. Solve for \(x\):
   a) \(3x - (x + 4) - 5 = 5(x - 4) - 4\)
   b) \(3x - 5x + 6 = 0\)
   c) \(x^2 - 5x - 14 = 0\)

6. Evaluate expressions:
   If \(x = -3\), evaluate \(x^2 - 2x - 1\)

7. Graph by HAND and on your GRAPHING CALCULATOR*
   a) \(4x + 3y = -12\)
   b) \(y = x^2 - 5x - 14\)

8. Find the equation of the line passing through 2 given points:
   \((2, -1)\) \((-1, -7)\)
9. Solve a system of equations by all of the following methods: substitution, elimination by addition (linear combinations), and graphically.

Given: \[
\begin{align*}
2x + y &= -3 \\
3x + 4y &= -2
\end{align*}
\]

10. Solve a first degree inequality in one variable:

Given: \[8 - 5x \geq 3x + 9\], solve for \(x\)

**Answers**

1. -2
2. a) \(7x^2 - 11xy + 2y\)  
   b) \(\frac{3d^8}{2b^9}\)
3. a) \(18x^2 - 9x - 35\)  
   b) \(4x^2 - 12x + 9\)
4. a) \((x - 7)(x + 2)\)  
   b) \(3a^2b(2b^2 - 1)\)
5. a) \(x = 5\)  
   b) \(x = \frac{5y - 6}{3}\)  
   c) \(x = 7, x = -2\)
6. 14
7. a)  
   b) 

![Figure 1](image1.png)

![Figure 2](image2.png)

8. \(y = 2x - 5\)
9. \(x = -2, y = 1\)
10. \(x \leq -1/8\) or \(-1/8 \geq x\)

* Students with no graphing calculator experience should enroll concurrently in MTH 93.*
How many of these problems can you miss and still succeed in MTH 95?

Ideally, NONE.

These problems are just a sample of the larger number of skills that you should be familiar with **BEFORE** taking this course.

If some of these ideas are not familiar to you, you should consider enrolling in one of the prerequisite courses (MTH 65 or MTH 70 or ALC 60, 61, 62, or 63).

Below is a sample of some skills you should have **BEFORE** entering

**MTH 111 – College Algebra (MTH 111B or MTH 111C)**

1. What is the equation of a line with slope $m = \frac{1}{2}$ which passes through the point $(6, -4)$?

2. Write each of these inequalities using interval notation:
   a) $2 < x \leq 7$  
   b) $x > 1$  
   c) $5 > x \geq -3$

3. Find the x-intercepts, the y-intercepts and the vertex of $y = x^2 - 8x + 7$ then graph the equation.

4. Simplify these exponential expressions:
   a) $\left( \frac{2x^3 y^{-2} z^{-5}}{8x^{-5} y^{-3} z^7 y} \right)^{-2}$  
   b) $\left( \frac{x^2 y^4}{x^5 z^{-3}} \right)^{\frac{1}{2}}$

5. Given the points (0,2) and (2,18), find the equation for an exponential function of the form $f(t) = a \cdot b^t$ which passes through both points.

6. Find the inverse of the function $f(x) = 2x - 5$. 
7. Given the function \( y = f(x) \) in Figure 1, find the domain and range of the function. What is the value of \( f(1) \)? Estimate the horizontal and vertical intercepts.

![Figure 1: \( y = f(x) \)](image_url)

8. Given \( f(x) = x - \sqrt[3]{x} \) and \( g(x) = \frac{3x + 2}{x} \), evaluate the composition \( (f \circ g) \left( \frac{2}{5} \right) \).

9. Find the value of \( f(g(2)) \) from the table below. For the function \( h \), which function type best describes its graph: linear; quadratic, or exponential?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
<th>( h(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
<td>7</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>-14</td>
<td>11</td>
<td>8</td>
</tr>
</tbody>
</table>
Answers

1. \( y = -\frac{1}{2}x - 1 \)
2) (a) (2,7] (b) (1,\( \infty \)) (c) \([-3,5]\)

3. \( x \)-intercepts: (1,0), (7,0) \( y \)-intercept: (0,7) Vertex: (4, 9)

\[
\begin{array}{l}
\text{Figure 2: } y = x^2 - 8x + 7
\end{array}
\]

4. (a) \( \frac{16z^4}{x^{16}} \) (b) \( \frac{y^7z^{10}}{x^{12}} \)
5. \( f(t) = 2 \cdot 3^t \)
6. \( f^{-1}(x) = \frac{x + 5}{2} \)

7. Domain: \((-\infty, 3]\) Range: \((-\infty, 4]\) \( f(1) = 2 \)
   Horizontal intercept is (1,0) Vertical intercept is \approx (0,1.8)

8. \( (f \circ g)(\frac{2}{5}) = 6 \)
9. \( f(g(2)) = -5 \) \( h \) is quadratic

How many of these problems can you miss and still succeed in MTH 111B or MTH 111C?

Ideally, NONE.

These problems are just a sample of the larger number of skills that you should be familiar with BEFORE taking this course.

If some of these ideas are not familiar to you, you should enroll in the prerequisite course (MATH 95).
Below is a sample of some skills you should have BEFORE entering

**MTH 112 – Elementary Functions (Trigonometry)**

You **MAY** use a calculator.

1. Find the inverse function for $f(x) = 2^{3-x}$.

2. If an initial sample of 50 mg of a radioactive substance decays to 40 mg in 235 years, find the half-life of the substance.

3. Solve $\ln x + \ln (x-2) = 3$.

4. On January 1, 1995, a park ranger estimates that there are 65 wolves in a wilderness area and that the wolf population is growing at an annual rate of 2.3%. When will there be 100 wolves in the area?

5. Draw a graph of a 5th degree polynomial with a negative leading coefficient, three single zeroes and a double zero.

6. Graph $g(x) = \frac{4-x^2}{x^2-9}$ and label all asymptotes and intercepts.

7. Given $h(x) = 2x^3 - 5x^2 - 14x + 8$, a) find intervals where $h$ is increasing and intervals where $h$ is decreasing. Solve for $x$ if $h(x) = 10$.

8. Solve for $x$ given the similar triangles shown in Figure 1.

9. Given the function $y = f(x)$ in Figure 2, graph the following transformations:
   a) $y = f(x) + 2$
   b) $y = f(x - 3)$
   c) $y = -2f(x)$

---

**Figure 1:** Triangles for #8

**Figure 2:** $y = f(x)$
10. From a common location, Car A heads north at 55 mph at the same time as Car B heads east at 45 mph. Assuming the roads are straight, how far apart are the two cars after 20 minutes?

Answers

1. \[ f^{-1}(x) = 3 - \log_2 x \]

2. The half life is almost 730 years (\(\approx 729.977\))

3. \(x \approx 5.592\)

4. There will be 100 wolves in December of 2013.

5.

![Figure 3: A Solution to #5](image)

![Figure 4: \(y = \dfrac{4 - x^2}{x^2 - 9}\)](image)

7. a) \(h\) is increasing on \((-\infty, -0.907) \cup (2.573, \infty)\)
   
   \(h\) is decreasing on \((-0.907, 2.573)\)

   b) \(h(x) = 10\) when \(x \approx -1.565, -0.152,\) or 4.216

8. \(x = 25.5cm\)

PCC COMPASS Placement Test Review Packet 73
9.

![Graph 1](image1)

![Graph 2](image2)

![Graph 3](image3)

10. The cars are approximately 23.688 miles apart in 20 minutes.

How many of these problems can you miss and still succeed in MTH 112?

Ideally, NONE.

These problems are just a sample of the larger number of skills that you should be familiar with BEFORE taking this course.

If some of these ideas are not familiar to you, you should consider enrolling in one of the prerequisite courses (MTH 111B or MTH 111C).

Below is a sample of some skills you should have BEFORE entering

**MTH 211 – Foundations of Elementary Math I**

You **MAY** use a calculator.

1. The temperature at 10:00 pm in West Yellowstone was 5 degrees below zero. By 3:00am the temperature had dropped 8 degrees. What was the temperature at 3:00am?
   a) $-3^\circ$    b) $3^\circ$    c) $12^\circ$    d) $-13^\circ$    e) $13^\circ$

2. What is the equation of a line with slope $-\frac{1}{2}$ which passes through the point (6,-4)?
   a) $6x + 4y = -\frac{1}{2}$    b) $y = -\frac{1}{2}x - 1$    c) $2x - 4y = 2$    c) $y = 6x - \frac{1}{2}$
3. A roast is to be cooked 20 minutes per pound. If the roast weighs 6 pounds and the cook wants it to finish cooking by 5:30pm, what is the latest time he can begin cooking the roast?
   
   a) 11:30am  b) 2:30pm  c) 3:30pm  d) 4:00pm  e) 4:10pm

4. If $x + 2y = 6$, then $2x + 4y = ?$
   
   a) 6   b) 8   c) 9   d) 10   e) 12

5. Les saved $8 on the purchase of a tire whose regular price was $40. What percent of the regular price did he save?
   
   a) 5%  b) 8%  c) 12%  d) 20%  e) 32%

6. The acceleration $A$ that results when force $F$ is applied to a body of mass $M$ can be calculated from the formula $F = MA$. What is the value of $A$ if $M = 1200$ and $F = 90,000$?
   
   a) 75   b) 750   c) 7500   d) 1,080,000   e) 108,000,000

7. If $\frac{4}{x} = 8$, then $x - 1 = ?$
   
   a) $-1\frac{1}{2}$   b) $-\frac{2}{3}$   c) $-\frac{1}{2}$   d) $\frac{1}{2}$   e) 1

8. Consider the problem: “Frank’s average speed riding a bicycle is 4 miles per hour less than twice Liz’s. If Frank’s average speed is 12 miles per hour, what is Liz’s average speed?”
   
   If $s$ represents Liz’s average speed riding a bicycle, which of the following equations can be used to solve the problem.
   
   a) $4 - 2s = 12$  b) $2s + 4 = 12$  c) $2s - 4 = 12$
   
   d) $s = 2(12) - 4$  e) $s = 2(12) + 4$
9. If $a = -2$, then the value of $4a^2 - 2a + 3$ is

   a) $-65$  
   b) $-17$  
   c) $15$  
   d) $23$  
   e) $71$

10. If $y = x^3$ and $x = \frac{1}{4}$, then what is the value of $y$?

   a) $\frac{1}{64}$  
   b) $\frac{1}{16}$  
   c) $\frac{1}{12}$  
   d) $\frac{1}{4}$  
   e) $\frac{3}{4}$

11. Given the function $y = f(x)$ in Figure 1, find the domain and range of the function. What is the value of $f(1)$? Estimate the horizontal and vertical intercepts.

   ![Figure 1](image_url)

   **Answers**

   1. d  
   2. b  
   3. c  
   4. e  
   5. d  
   6. a  
   7. c  
   8. c  
   9. d  
   10. a

   11. Domain: $(-\infty, 3\}$  
   Range: $(-\infty, 4\}$  
   $f(1) = 2$.

   Horizontal intercept is 1, vertical intercept is 1.8.

How many of these problems can you miss and still succeed in MTH 211?

Ideally, NONE.

These problems are just a sample of the larger number of skills that you should be familiar with **BEFORE** taking this course.

If some of these ideas are not familiar to you, you should enroll in the prerequisite course (MTH 95).
Below is a sample of some skills you should have **BEFORE** entering

**MTH 241 – Calculus for Business, Life Science, and Social Science**

You **MAY** use a calculator.

1. Solve for $x$:
   \[
   \frac{x^2 + 3x}{x^2 + 2x - 8} \geq 0
   \]

2. Solve for $x$:
   \[
   4 < \left| \frac{2}{3}x + 5 \right|
   \]

3. What is the domain, range, and graph of the function
   \[
   y = f(x) = \begin{cases} 1 - x, & \text{if } x < 0 \\ 1, & \text{if } x > 0 \end{cases}
   \]

4. Solve the system:
   \[
   \begin{align*}
   y &= \frac{18}{x + 4} \\
   x - y + 7 &= 0
   \end{align*}
   \]

5. Solve for $x$:
   \[
   \log_x(2x + 3) = 2
   \]

6. Solve for $x$:
   \[
   e^{\ln(x+4)} = 7
   \]

7. If $2600$ is invested for 6.5 years at 6% interest compounded quarterly, find:
   a) The compounded amount 
   b) The compounded interest

8. If $f(x) = 4x$ and $g(x) = x^2 + 6x^{-1}$, find:
   a) $(f - g)\left(\frac{x}{2}\right)$ 
   b) $(fg)(-0.5)$ 
   c) $\left[ g\left(\frac{1}{x}\right) \right]$

9. Find the effective interest rate equivalent to an annual rate of 6 percent compounded continuously.

10. Give the domain, range, and sketch the graph of the function:
    \[
    y = f(x) = \sqrt{x - 2}
    \]
Answers

1. \( x < -4 \text{ or } -3 \leq x \leq 0 \text{ or } x > 2 \)

\((-\infty, -4) \cup \left\{ -3, 3 \right\} \cup (2, +\infty)\)

2. \[
\begin{cases} 
  x < -\frac{27}{2} \text{ or } x > -\frac{3}{2} \\
  (-\infty, -\frac{27}{2}) \cup (-\frac{3}{2}, +\infty)
\end{cases}
\]

3. Domain: \((-\infty, 0) \cup (0, \infty)\)
Range: \([0, \infty)\)

4. \((-1, 6), (-10, -3)\)

5. \(x = 3\)

6. \(x = 3\)

7. a) $3829.04 \text{ b) } $1229.04$

8. a) \(2x - \frac{x^2 - 12}{4} \text{ b) } 23.5\)
\(c) \frac{4}{x^2} + 24x\)

9. 6.18%

10. Domain: \([1, \infty)\)
Range: \([1, \infty)\).

How many of these problems can you miss and still succeed in MTH 241?

Ideally, NONE.

These problems are just a sample of the larger number of skills that you should be familiar with \textbf{BEFORE} taking this course.

If some of these ideas are not familiar to you, you should consider enrolling in one of the prerequisite courses (MTH 111B or MTH 111C).
Below is a sample of some skills you should have **BEFORE** entering

**MTH 243 – Statistics I**

1. If \( z = \frac{a-b}{c} \), solve for \( a \)

2. If \( \frac{m}{\sqrt{n}} = 3 \), solve for \( n \)

3. Using mental math only (no calculator, no pencil & paper), evaluate:
   a) \( \frac{a-b}{c} \), when \( a=14 \), \( b=13 \), \( c=8 \), and \( n=4 \)
   b) \( \frac{x-np}{\sqrt{np(1-p)}} \) when \( x=16 \), \( n=100 \), and \( p=0.2 \)
   c) \( \frac{1}{2} + \frac{1}{4} - \frac{1}{8} \)

4. Refer to the scatter plot in Figure 1.
   a) Find the slope of the line.
   b) Write an equation of the line.
   c) Interpret the slope in the context of the data.

![Figure 1: \( y \) is the number of units produced per week by an employee who has been on the job for \( x \) years](image-url)
5. Use your calculator to evaluate: (round each result to 3 significant digits)
   a) \(0.463 \pm 1.96 \sqrt{\frac{(0.463)(0.537)}{423}}\)
   b) \((-1 - 1.171)^2 \cdot (2) + (0 - 1.171)^2 \cdot (3) + (1 - 1.171)^2 \cdot (5)\)
   c) \(\frac{51,800 - 55,000}{4500} \div \sqrt{8}\)
   d) \(f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \) at \(x = 0\)

6. Given \(f(x) = x - 20\), find:
   a) the \(x\)- and \(y\)-intercepts of the graph of \(y = f(x)\)
   b) \(f^{-1}(-4)\)

7. A nicotine patch or a placebo patch was randomly assigned to each of 240 smokers who expressed a desire to quit. Here are the numbers who had quit and not quit smoking after 8 weeks of wearing the patches.

<table>
<thead>
<tr>
<th>Smoking after 8 weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
</tr>
<tr>
<td>Nicotine Patch</td>
</tr>
<tr>
<td>Placebo Patch</td>
</tr>
</tbody>
</table>

   a) What proportion of the subjects in the study quit smoking after 8 weeks?
   b) What proportion of the nicotine patch users quit after 8 weeks?

**Answers**

1. \(a = b + cz\)
2. \(n = \frac{z^2 \cdot m^2}{3^2}\)
3. a) \(\frac{1}{4}\)   b) \(-1\)   c) \(\frac{5}{8}\)
4. a) The slope is \(\frac{3}{7}\)   b) The equation is: \(y = \frac{3}{7}x + 2\)
c) The mean weekly production increases at a rate of \( \frac{3}{7} \) units per week per year on the job.

5. a) .415, .511 b) 2.60 c) –2.01 d) .399

6. a) (20, 0), (0, -20) b) 16

7. a) .333 approx b) .467 approx.

How many of these problems can you miss and still succeed in MTH 243?

Ideally, NONE.

These problems are just a sample of the skills that you should be familiar with BEFORE taking this course.

If some of these ideas are not familiar to you, you should enroll in one of the prerequisite courses (MTH 111B or MTH 111C).

Below is a sample of some skills you should have BEFORE entering

**MTH 251 – Calculus I**

You **MAY NOT** use a calculator.

1. Simplify the expression \( \frac{(x + h)^2 - x^2}{h} \) so \( h \) does not appear in the denominator.

2. Answer each question in reference to the function \( y = f(x) \) shown in Figure 1.

   a) What is the value of \( f(2) \)?

   b) Evaluate the expression \( \frac{f(0) - f(-4)}{0 - (-4)} \)

3. Given the functions \( f(x) = x + 2 \) and \( g(x) = 4 - x^2 \), find the functions \( (g \circ f)(x) \) and \( (f \circ g)(x) \).

4. Find the vertical and horizontal asymptotes of \( f(x) = \frac{x + 2}{x^2 - 1} \).
5. Given the function \( g(x) = -x^2 + 3 \), find the equation of the line that intersects the graph of \( g(x) \) at \( x = 0 \) and \( x = 4 \).

6. Which of the following expressions is equal to the length of side \( c \) in the triangle shown in Figure 2?

   a) \( \tan(\phi) \)  
   b) \( \cos(\phi) \)  
   c) \( \cos(\phi) \)  
   d) \( \sin(\phi) \)  
   e) none of these

   ![Figure 2: Triangle for #6](image)

7. The graph of the function \( f(x) = x^3 - 3x \) is shown in Figure 3.
   a) On what intervals is the function increasing?
   b) Over what intervals is the function decreasing?
   c) On the interval \((-1.8, 1.8)\) what is the maximum value of \( f(x) \)?

   ![Graph of the function \( f(x) = x^3 - 3x \) for problem 7.](image)

8. Expand and simplify completely: \( \ln\left(\frac{3e^x}{x\sqrt{x+1}}\right) \)

9. Solve for \( x \): \( 2\sin(x)\cos(x) = 0 \) on the interval \( 0 \leq x < 2\pi \)
10. Simplify: \( \frac{x^2 + 8x + 15}{x + 3} \)

11. Find the inverse function of \( f(x) = \frac{x}{x + 4} \)

**Answers**

1. \( 2x + h \), for \( h \neq 0 \)
2. a) \( f(2) = 2 \) \hspace{1cm} b) 1
3. \((g \circ f)(x) = -x^2 - 4x\) and \((f \circ g)(x) = -x^2 + 6\)
4. Vertical: \( x = 1 \), \( x = -1 \) \hspace{1cm} Horizontal: \( y = 0 \)
5. \( y = -4x + 3 \)
6. e
7. a) \( (-\infty, -1) \cup (1, \infty) \) \hspace{1cm} b) \( (-1,1) \) \hspace{1cm} c) 2
8. \( \ln x + x - \ln x - \frac{1}{2} \ln x + 1 \)
9. \( \left\{ 0, \frac{\pi}{2}, \frac{3\pi}{2} \right\} \)
10. \( x + 5 \), for \( x \neq -3 \)
11. \( f^{-1}(x) = \frac{4x}{1-x} \)

How many of these problems can you miss and still succeed in MTH 251?

Ideally, NONE.

These problems are just a sample of the larger number of skills that you should be familiar with **BEFORE** taking this course.

If some of these ideas are not familiar to you, you should enroll in one of the prerequisite courses (MTH 111C or MTH 112).
Below is a sample of some skills you should have BEFORE entering

**MTH 252 – Calculus II**

You **MAY NOT** use a calculator.

1. **LIMITS:**
   a) \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) if \( f(x) = x^2 \)
   b) \( \lim_{x \to 2} \frac{x^2 - 4}{x - 2} \)
   c) \( \lim_{x \to 2} \frac{x - 1}{x - 2} \)
   d) \( \lim_{x \to \infty} \frac{\sqrt{x^2 + 2}}{3x - 6} \)
   e) \( \lim_{x \to 3} f(x) \) if \( f(x) = \begin{cases} \sqrt{x+13}, & (\infty, 3) \\ x^2 - 5, & [3, \infty) \end{cases} \)
   f) \( \lim_{\theta \to \pi^-} \csc \theta \)
   g) \( \lim_{t \to 1^+} e^t \)
   h) \( \lim_{x \to 0^+} \ln \theta \)
   i) \( \lim_{y \to \infty} \tan^{-1} \theta \)

2. **DERIVATIVES:** Find \( \frac{dy}{dx} \)
   a) \( y = 4\sqrt{x} + \frac{1}{\sqrt{x}} \)
   b) \( y = \frac{(2x - 1)^6}{(3x + 4)^5} \)
   c) \( y = \ln(3x^2 + 2x) \)
   d) \( y = \cos(5x) - \sin^2(x) \)
   e) \( x^2 + 3xy - 5y^2 = 9 \)

3. **A DISCONTINUITY** exists for what values of \( x \)?
   a) \( \frac{3x + 1}{x^2 + 7x - 2} \)
   b) \( \cot(x) \)

4. **GIVEN:** \( f(x) = \frac{1}{12} x^4 + \frac{1}{6} x^3 - 3x^2 \)
   a) What are the CRITICAL VALUES?
   b) For what \( x \)-values is the curve CONCAVE down?
5. Find the local extrema of \( g(x) = \frac{1-\ln(x)}{x^2} \)

6. For what \( x \)-values is there a local maximum or minimum of \( y = \frac{x^2 - 1}{x^3} \)?

7. What will be the RATE OF CHANGE in the area of a circle when the diameter is 20 feet if the radius is decreasing 1/10 foot per second?

\[ \text{Answers} \]

1. a) \( 2x \) b) \( 4 \) c) \( \infty \) d) \( \frac{1}{3} \) e) \( 4 \)
   f) \( \infty \) g) \( 0 \) h) \( -\infty \) i) \( \frac{\pi}{2} \)

2. a) \( \frac{2}{\sqrt{x}} - \frac{1}{2x^2} \) b) \( \frac{3(2x-1)^3(2x+1)}{(3x+4)^6} \)
   c) \( \frac{6x+2}{3x^2 + 2x} \) d) \( -5\sin(5x) - 2\sin(x)\cos(x) \)
   e) \( \frac{2x + 3y}{10y - 3x} \)

3. a) \( \frac{-7 \pm \sqrt{57}}{2} \) b) \( k\pi \)

4. a) \( 0, -\frac{3 \pm 3\sqrt{33}}{4} \) b) \( -3 < x < 2 \)

5. \( \left( e^{3/2}, -\frac{1}{2e^{3}} \right) \) is a local minimum point

6. A local maximum occurs at \( x = \sqrt{3} \)
   A local minimum occurs at \( x = -\sqrt{3} \)

7. \( \frac{dA}{dt} = -2\pi \frac{\text{ft}^2}{\text{sec}} \)
How many of these problems can you miss and still succeed in MTH 252?

Ideally, NONE.

These problems are just a sample of the larger number of skills that you should be familiar with BEFORE taking this course.

If some of these ideas are not familiar to you, you should enroll in the prerequisite course (MTH 251)

Below is a sample of some skills you should have BEFORE entering

**MTH 253 – Calculus III**

You **MAY NOT** use a calculator.

1. Graph and find the area between the curves $y = 4\sqrt{x}$ and $2y = x^2$.

2. Simplify the expression $\frac{f(x) + 1}{f(x)}$ for each function.
   a) $f(x) = \frac{3 + 2^{n+1}}{7^{n-2}}$  
b) $f(x) = \frac{n}{n + 1}$

3. Evaluate each improper integral.
   a) $\int_{-\infty}^{\infty} e^{-x} \, dx$  
   b) $\int_{1}^{\infty} \frac{2 \, dt}{1 + t^2}$

4. Evaluate each limit; use L’Hopital’s Rule where appropriate.
   a) $\lim_{x \to \infty} \frac{\sin(x)}{x}$  
   b) $\lim_{t \to \infty} \left( e^{-t^2} \right)$  
   c) $\lim_{\theta \to \infty} \left( 3\theta \tan\left( \frac{5}{\theta} \right) \right)$

5. Find the velocity, speed and acceleration at $t = 1$ if $s = 2t^3 - 5t$

6. Determine the interval(s) over which the function $g(t) = \frac{t^3 - 5t^2 - 14t + 8}{3}$ is decreasing.
7. Integrate:  
   a) \[ \int \frac{1}{x} \cos \sqrt{x} \, dx \]  
   b) \[ \int \sqrt{x} \sec^2 x \, dx \]  
   c) \[ \int e^{3x} \, dx \]

8. Find: \[ \sum_{k=1}^{10} k \]

9. Which of the following expressions is equal to the length of side c in the triangle shown in Figure 1?

   a) \[ a \tan (\phi) \]  
   b) \[ b \cos (\phi) \]  
   c) \[ a \cos (\phi) \]  
   d) \[ b \sin (\phi) \]  
   e) none of these

![Figure 1: Triangle for #9](image.png)

Answers

1. \[ \frac{32}{3} \]
2. a. \[ \frac{25}{7} \]  
   b. \[ \frac{x^2 + 2x + 1}{x^2 + 2x} \]
3. a. \[ \frac{1}{e} \]  
   b. \[ \frac{\pi}{2} \]
4. a. 0  
   b. 0  
   c. 15

5. The speed and velocity are both 1 and the acceleration is 12

6. \[ 2, 7 \]

7. a. \[ 2 \sin (x) + C \]  
   b. \[ \frac{2}{3} \tan^{1/2} + C \]  
   c. \[ \frac{x^4}{4} + C \]

8. \[ 55 \]

9. \[ e \]

How many of these problems can you miss and still succeed in MTH 253?

Ideally, NONE.

These problems are just a sample of the larger number of skills that you should be familiar with BEFORE taking this course.

If some of these ideas are not familiar to you, you should enroll in the prerequisite course (MTH 252).
Math Review – Part II
Refreshing your Math

Please note that the following information is meant for review only. If the material or part of it is unfamiliar to you, it is recommended that you take the corresponding math class. Under each topic you will find the math class in which that particular material is taught.

Multiplication can be symbolized in different ways. For example: 5 • 2, 5 x 2, or (5)(2). The use of “x” is not useful for algebra and will therefore not be used here. The other two variations will be used interchangeably. If variables are used multiplication is assumed if no sign appears. For example: 3a or ab.

**Integers**
(Math 20)

**Definitions**
Integers are counting numbers, their negative counterparts, and zero:

```
-5   -4   -3   -2   -1
  | 0  | 1  | 2  | 3  | 4  | 5 |
```

The distance of a number from zero is called the *absolute value*. The absolute value is always positive:  \( |5| = 5 \) and  \( |-5| = 5 \)
Multiplying, Dividing, Adding, and Subtracting Integers

Examples:
12 ÷ 4 = 3  
-2 • (-3) = 6  
8 ÷ (-2) = -4  
-5 • 6 = 30  
4 + 9 = 13  
-6 + (-11) = -17  
9 + (-7) = 2  
-14 + 6 = -8  
9 - 12 = 9 + (-12) = -3  
-14 - 7 = -14 + (-7) = -21  
15 - (-3) = 15 + 3 = 18  
-4 - (-5) = -4 + 5 = 1

Practice Problems:
1. 10 • (-7)  
2. -8 • (-5)  
3. -3 • (-15)  
4. (-1)(15)  
5. (0)(-8)  
6. 80 ÷ (-10)  
7. -63 ÷ 7  
8. -81 ÷ (-9)  
9. 0 ÷ (-5)  
10. -7 ÷ 0  
11. -3 + (-8)  
12. 10 + (-4)  
13. 5 + (-9)  
14. -7 + 2  
15. -6 + 8  
16. 8 - 13  
17. -7 - 10  
18. 12 - (-4)  
19. -5 - (-1)  
20. -9 - (-9)

Answers to Practice Problems:
1. -70  
2. 40  
3. 45  
4. -15  
5. 0  
6. -8  
7. -9  
8. 9  
9. 0  
10. undefined  
11. -11  
12. 6  
13. -4  
14. -5  
15. 2  
16. -5  
17. -17  
18. 16  
19. -4  
20. 0
Fractions
(Math 20)

Definitions

Fraction = \( \frac{\text{Numerator}}{\text{Denominator}} \)

When the numerator is smaller than the denominator we call the fraction *proper*. If the numerator is greater than the denominator we call the fraction *improper*. Improper fractions can be written as *mixed numbers*, which is as an addition of a whole number and a proper fraction. For example:

\( \frac{2}{3} \) is a proper fraction;

\( \frac{4}{3} \) is an improper fraction and can be written as a mixed number: \( 1 \frac{1}{3} \)

Whole Numbers such as 5 can be written as \( \frac{5}{1} \)

The *reciprocal* of a fraction has the numerator and denominator switched. For example:

\( \frac{3}{2} \) is the reciprocal of \( \frac{2}{3} \)

Mixed Numbers

Mixed numbers can be converted to improper fractions like this:

\( \frac{3}{5} = \frac{3 \cdot 5 + 4}{5} = \frac{19}{5} \)

Improper fractions can be converted to mixed numbers by dividing with remainder:

19 ÷ 5 = 3 R 4 which translates into \( 3 \frac{4}{5} \)

Simplifying Fractions

When simplifying fractions we divide the numerator and the denominator by a common factor. Like this:

\( \frac{28}{48} = \frac{28 ÷ 4}{48 ÷ 4} = \frac{7}{12} \)
This can also be done in several steps:

\[
\frac{28}{48} = \frac{28 \div 2}{48 \div 2} = \frac{14}{24} = \frac{14 \div 2}{24 \div 2} = \frac{7}{12}
\]

At the end of a calculation fractions should always be simplified.

**Multiplying, Dividing, Adding, and Subtracting Fractions**

**Multiplying**
- if possible simplify fractions
- multiply numerators
- multiply denominators
- if needed simplify again

**Dividing**
- multiply the first fraction with the reciprocal of the second fraction

**Adding and Subtracting**
- find a common denominator
- expand each fraction to that common denominator
- add the numerators
- keep the common denominator

**Examples:**

\[
\begin{align*}
\frac{5}{3} \cdot \frac{9}{20} &= \frac{45}{60} = \frac{3}{4} \\
\frac{5}{3} \div \frac{7}{15} &= \frac{5 \cdot 15}{3 \cdot 7} = \frac{75}{21} = \frac{7}{3} \\
\frac{2}{3} + \frac{1}{7} &= \frac{2 \cdot 7}{3 \cdot 7} + \frac{1 \cdot 3}{7 \cdot 3} = \frac{17}{21}
\end{align*}
\]

or

\[
\begin{align*}
\frac{1\frac{5}{3}}{1\frac{3}{4}} &= \frac{1 \cdot 3 + 5}{1 \cdot 4 + 3} = \frac{3}{4} \\
\frac{5}{3} \div \frac{7}{15} &= \frac{5 \cdot 15}{3 \cdot 15} = \frac{75}{45} = \frac{7}{3} \\
\frac{5}{6} - \frac{1}{8} &= \frac{5 \cdot 4 - 1 \cdot 3}{24} = \frac{20 - 3}{24} = \frac{17}{24}
\end{align*}
\]

**Practice Problems:**

1. \(\frac{3}{4} \cdot \frac{5}{11}\)  
2. \(\frac{-2}{5} \cdot \frac{3}{7}\)  
3. \(\frac{-7}{9} \cdot \frac{-3}{5}\)  
4. \(\frac{-5}{21} - \frac{14}{-25}\)  
5. \(\frac{9}{6} \cdot \frac{5}{6}\)  
6. \(\frac{1}{5} \div \frac{3}{4}\)  
7. \(\frac{-2}{5} \div \frac{3}{4}\)  
8. \(\frac{-7}{10} - \frac{-5}{9}\)  
9. \(\frac{8}{15} \div \frac{2}{5}\)  
10. \(\frac{-4}{5} \div \frac{4}{4}\)
11. $\frac{2}{3} + \frac{1}{4}$  
12. $\frac{-1}{5} + \frac{-3}{10}$  
13. $\frac{7}{9} + \frac{-1}{6}$  
14. $\frac{5}{8} - \frac{-1}{12}$  
15. $\frac{5}{12} - \frac{-3}{10}$

Answers to Practice Problems:

1. $\frac{15}{44}$  
4. $\frac{2}{15}$  
7. $\frac{-8}{15}$  
10. $\frac{-1}{5}$  
13. $\frac{11}{18}$  

2. $\frac{-6}{35}$  
5. $\frac{15}{2} = \frac{7}{2}$  
8. $\frac{72}{50} = \frac{1}{50}$  
11. $\frac{11}{12}$  
14. $\frac{17}{24}$  

3. $\frac{21}{45}$  
6. $\frac{4}{15}$  
9. $\frac{4}{3} = \frac{1}{3}$  
12. $\frac{-1}{2}$  
15. $\frac{43}{60}$

Order of Operations
(Math 20)

When evaluating numerical expressions we follow the Order of Operations:

1. Evaluate the inside of the parentheses or other grouping symbols first.  
   Grouping symbols include also brackets, absolute value, square roots, and 
   complex numerators and denominators.
2. Evaluate exponents.
3. Multiply or divide, whichever comes first as you read left to right.
4. Add or subtract, whichever comes first as you read left to right.

Example 1:
$25 - (2 + 4)^2 \div 4 \cdot 2 + 1$  
Evaluate inside of parantheses first.  
$= 25 - 6^2 \div 4 \cdot 2 + 1$  
Evaluate exponents next.  
$= 25 - 36 \div 4 \cdot 2 + 1$  
Divide first since the division is further left than the multiplication  
$= 25 - 9 \cdot 2 + 1$  
Multiply.  
$= 25 - 18 + 1$  
Subtract first since the subtraction is further left than the addition.  
$= 7 + 1$  
Add.  
$= 8$
Example 2:
\[ 25 - (2 - 8)^2 + (-8) \cdot (-\frac{5}{3}) + \frac{1}{2} \]
Evaluate inside of parantheses first.
\[ = 25 - (-6)^2 + (-8) \cdot (-\frac{5}{3}) + \frac{1}{2} \]
Evaluate exponents next.
\[ = 25 - 36 + (-8) \cdot (-\frac{5}{3}) + \frac{1}{2} \]
Divide first since the division is further left than the multiplication.
\[ = 25 + \frac{3}{2} \cdot (-\frac{5}{3}) + \frac{1}{2} \]
Multiplication next by first simplifying the fractions.
\[ = 25 - \frac{15}{2} + \frac{1}{2} \]
Multiply.
\[ = \frac{50}{2} - \frac{15}{2} + \frac{1}{2} \]
Subtract first since the subtraction is further left than the addition.
\[ = \frac{35}{2} + \frac{1}{2} \]
The common denominator is 2.
\[ = \frac{36}{2} \]
Add.
\[ = 18 \]
Simplify.

Practice Problems:
1. \[ 10 - (9 - 2 \cdot 2)^2 \div 5 + 3 \]
2. \[ \frac{2}{3} - \frac{1}{3} \div \frac{2}{3} \cdot \frac{5}{3} \]
3. \[ 7^2 - 5 \cdot 8 + \frac{4 + 3 \cdot 2^3}{4 \cdot 5 - 4(4 - 1)} \]
4. \[ 25 - 36 \div 3^2 \cdot 2^2 + 24 \div 2 \cdot 3 - (5 \cdot (-6)) \]
5. \[ (36 - 4^2 \div 2 \cdot 2)^2 - (-5 - 30 \div 2 \cdot 3 + 40)^2 \]
6. \[ \frac{10}{3} \div \frac{15}{27} - \frac{36}{45} \div \frac{48}{36} \cdot \frac{30}{24} \]

Answers to Practice Problems:
1. \[ 8 \]
2. \[ \frac{23}{2} = 11 \frac{1}{2} \]
3. \[ \frac{25}{2} = 12 \frac{1}{2} \]
4. \[ 75 \]
5. \[ 300 \]
6. \[ -\frac{27}{4} = -6 \frac{3}{4} \]
Solving Linear Equations
(Math 60)

Definitions

A variable is a place holder for a number. It is represented by a letter.
Example: x, y, a, b

A term is a number, variable, or a combination of both if multiplied together. Different terms are separated by addition and subtraction.
Example: In the expression 5 + 7x – 7(x+2) the terms are 5 and 7x, and -7(x+2).

Like terms are terms that have the same variables with the same exponents. In an equation like terms can be combined.
Example: 7x and 2x are like terms, 8x and 4x² are not like terms.

Distributive property: a(x+y) = ax + ay
Example: 2(3x-4) = 6x – 8

The Golden Rule of Algebra

What you do to one side of an equation or inequality you have to do to the other side of the equation or inequality as well.

The objective for solving equations or inequalities is to isolate the variable on one side of the equation or inequality. To achieve that we can do a combination of the following operations ("something" can be a number, variable, or a combination of both):

- Add something to both sides.
- Subtract something from both sides.
- Multiply something to both sides.
- Divide both sides by something.
- Square both sides.
- Take the square root of both sides.

In case of an inequality, if multiplied or divided by a negative number the sign will turn around (for instance from < to >).
Example 1:
\[2x - 8 = 7x + 2\]
\[2x - 8 - 7x = 7x + 2 - 7x\] subtracting 7x from both sides to bring variables to the same side
\[-5x - 8 = 2\] combining like terms
\[-5x - 8 + 8 = 2 + 8\] adding 8 to both sides to isolate variable
\[-5x = 10\] combining like terms
\[-\frac{5x}{-5} = \frac{10}{-5}\] dividing both sides by -5 to isolate variable
\[x = -2\] simplify fractions

Example 2:
\[7a - (a-1) + 8(a - 4) < 3(7a + 12) + 3\]
\[7a - a + 1 + 8a - 32 < 21a + 36 + 3\] distributing
\[14a - 31 < 21a + 39\] simplifying like terms
\[14a - 31 - 21a < 21a + 39 - 21a\] subtracting 21a from both sides to bring variables to the same side
\[-7a - 31 < 39\] combining like terms
\[-7a - 31 + 31 < 39 + 31\] adding 31 to both sides to isolate variable
\[-7a < 70\] combining like terms
\[-\frac{7a}{-7} > \frac{70}{-7}\] dividing both sides by -7, turning around the inequality sign
\[a > -10\] simplify fractions

Practice Problems:
Solve each equation or inequality.

1. \[5x - 3 + 2x = 15 + 3x + 2\]
2. \[9b - 8 + 8b > 17 + 2b + 5\]
3. \[41y - 53 + 38y = 46 + 73y + 81\]
4. \[4 + 7a - 11 = 24a - 6 - 13a\]
5. \[2 + 8z - 5 < 8z - 9 - 4z\]
6. \[54 + 79k - 91 = 34k - 37 + 45k\]
7. \[15 - 3(b+7) = 2(b+2)\]
8. \[23(x+1) + 7(2x-1) = 43x - 6(x-2)\]
9. \[4n - 7(n-5) +10 < 8 - 15(n+2) - 6x\]
10. \[54x + 6(7x-4) \geq 7(8x+7) - 9(4x-8x)\]

Answers to Practice Problems:
1. \[x = 5\]
2. \[b > 2\]
3. \[y = 30\]
4. \[x = -\frac{1}{4}\]
5. \[x < -\frac{3}{2}\]
6. all real numbers
7. \[x = -2\]
8. no solutions
9. \[x < -\frac{67}{18}\]
10. \[x \geq \frac{73}{4}\]
Graphing Lines
(Math 60)

Definitions

A line is the graphic representation of a linear equation in two variables. Example: The linear equation $y = 2x + 1$ can be graphically represented as:

The slope is a measure of how steep the line is.

The $x$-intercept is the intersection of the line and the x-axis.
The $y$-intercept is the intersection of the line and the y-axis.

If the line is in the form $y = mx + b$ we call it the slope-intercept form. With $m$ representing the slope and $b$ representing the $y$-intercept $(0,b)$. If the line is in the form $ax + bx = c$ we call it the standard form.

We get the slope-intercept form from the standard form by solving for $y$.
We get the standard form from the slope-intercept form by subtracting $mx$ from both sides (add if $m$ is negative) and multiply by the common denominator (if there are fractions).

Graphing a Line

To graph a line in slope-intercept form we make a table of values by choosing several values for $x$ and solving the equation for $y$ respectively.
Example: For the equation \( y = -\frac{1}{2} x - 1 \) we choose 0, 2, and -2 for our x values (2 was chosen so the fraction simplifies easily). We then substitute these values for x and solve for y. In the case of x=2 this is how:

\[
y = -\frac{1}{2} \cdot 2 - 1
\]

Finding the other points the same way we get the following table:

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0 & -1 \\
2 & -2 \\
-2 & 0 \\
\hline
\end{array}
\]

Graphing each of those points and connecting the dots, we get the following graph:

\[
y = -\frac{1}{2}x - 1
\]

To graph a line in standard form we make a table of values by choosing 0 for x and solving the equation for y, and then choosing 0 for y and solving the equation for x. Example: For the equation \( x + 2y = -2 \) we choose 0 for x and y and then solve for the other variable respectively. For x=0 this is how:

\[
x + 2y = -2
\]

Finding the other points the same way we get the following table:

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0 & -1 \\
-2 & 0 \\
\hline
\end{array}
\]
Graphing each of those points and connecting the dots, we get the following graph:

![Graph showing points and connecting dots](image)

**Finding the Slope and Intercepts**

If the equation appears in slope-intercept form \( y = mx + b \) then \( m \) represents the slope and \( b \) is the \( y \)-intercept \((0,b)\).

Another way of finding the slope is by using two points from the line and the slope formula:

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

The \( y \)-intercept can also be found by choosing 0 (zero) for \( x \) and solving for \( y \).

The \( x \)-intercept can be found by choosing 0 (zero) for \( y \) and solving for \( x \).

**Example:** In the example of \( y = -\frac{1}{2}x - 1 \) we know the slope is \(-\frac{1}{2}\) and the \( y \)-intercept is \((0,-1)\). We find the \( x \)-intercept by choosing 0 for \( y \) and solving for \( x \):

\[
0 = -\frac{1}{2}x - 1
\]

\[
2(0) = 2(-\frac{1}{2}x - 1)
\]

\[
0 = 2(-\frac{1}{2}x) - 2(1)
\]

\[
0 = -x - 2
\]

\[
0 + x = -x - 2 + x
\]

\[
x = -2
\]
Practice Problems:
Graph each line and find its slope and intercepts.

1. \( y = x - 1 \)
2. \( y = -3x - 4 \)
3. \( y = \frac{1}{3}x + 2 \)
4. \( 2x + 3y = 5 \)
5. \( x - y = -2 \)
6. \( 3x - y = 4 \)

Answers to Practice Problems:

1. \( m=1 \)
   - \( x\)-intercept: (1,0)
   - \( y\)-intercept: (0,1)

2. \( m=-3 \)
   - \( x\)-intercept: \( \left(-\frac{4}{3}, 0\right) \)
   - \( y\)-intercept: (0,-4)

3. \( m= \frac{1}{3} \)
   - \( x\)-intercept: \( (-6,0) \)
   - \( y\)-intercept: (0,2)

4. \( m= -\frac{2}{3} \)
   - \( x\)-intercept: \( \left(\frac{5}{2},0\right) \)
   - \( y\)-intercept: \( \left(0, \frac{5}{3}\right) \)
5. \( m = 1 \)
   
   x-intercept: (-2,0)
   y-intercept: (0,2)

6. \( m = 3 \)
   
   x-intercept: \( \left( \frac{2}{3},0 \right) \)
   y-intercept: (0,-2)

Laws of Exponents
(Math 65)

When simplifying expressions with exponents we follow these laws:

\[
\begin{align*}
    a^m a^n &= a^{m+n} \\
    \frac{a^m}{a^n} &= a^{m-n} \\
    (a^m)^n &= a^{mn} \\
    a^{-m} &= \frac{1}{a^m} \\
    a^0 &= 1 \quad (a \neq 0) \\
    (ab)^m &= a^m b^m 
\end{align*}
\]

Examples:
1. \( x^5 \cdot x^3 = x^{5+3} = x^8 \)
2. \( \frac{h^6}{h^{14}} = h^{6-14} = h^{-8} = \frac{1}{h^8} \)
3. \[8^3 \cdot 4 = 8^{3+4} = 8^{12}\]

4. 
\[
\left( \frac{5^4 x^7 y^{13} z^5 y^{12}}{5^6 x^2 z^5} \right)^2 = 5^{4-6} x^{7-2} y^{13+12} z^{5-5} 2
\]
\[
= 5^{-2} x^5 y^{25} z^0 2
\]
\[
= 5^{-2} x^5 y^{25} \cdot 1 2
\]
\[
= 5^{-2} x^5 y^{25} 2
\]
\[
= 5^{2(-2)} x^{2(5)} y^{2(25)}
\]
\[
= 5^{-4} x^{10} y^{50}
\]
\[
= \frac{x^{10} y^{50}}{5^4}
\]

Practice Problems:
Simplify:
1. \[5^2 \cdot 5^4\]
2. \[\frac{x^{11}}{x^7}\]
3. \[(a^4)^3\]
4. \[c^{-7}\]
5. \[(yz)^6\]
6. \[
\frac{7^3 x^4 \cdot 7^2 x^3}{7^4 x^5}
\]

Answers to Practice Problems:
1. \[5^6\]
2. \[x^8\]
3. \[a^{20}\]
4. \[\frac{1}{c^7}\]
5. \[y^{6z^{18}}\]
6. \[7x^2\]
7. \[\frac{a^2 b^3}{a^5 b^{10}}\]
8. \[\frac{4^7 s^2 t^2}{4^4 s t^4}\]
9. \[\frac{3^2 x^3 y^4 z^{10} x^{12} y^{10} z^{13} x^6 z}{3^3 y^7 z^4}
\]
10. \[\frac{4^{-3} a b^{-4} c^5 \cdot 2^{-10} a^{-6} c}{2^{-8} a^{-12} \cdot 4^{-5} b^{-3} c^4}\]
Functions
(Math 60, 65, 95, 111)

Consider a function \( f(x) \). Then \( x \) is the input and \( f(x) \) the output. All eligible inputs make up the domain. All outputs make up the range.

Unless otherwise noted the domain is usually all real numbers. The two most common exceptions are:

1. If the function contains a fraction the domain will be restricted because the denominator cannot be zero. The function has a vertical asymptote at that point.
2. If the function contains an even root the domain will be restricted because the radicand has to be greater or equal to zero.

To find the range it is often helpful to graph the function by solving \( f(x) \) for as many \( x \) as needed to see what the function looks like.

**Example 1:** For the function \( f(x) = 2x + 1 \) the domain is all real numbers and so is the range.

**Example 2:** For the function \( f(x) = \frac{3}{5x+2} \) the domain is restricted by the fact that \( 5x+2 \) cannot be zero. \( 5x+2=0 \) when \( x = -\frac{2}{5} \). Therefore the domain is all real numbers with the exception of \( -\frac{2}{5} \). We can write that mathematically in different ways:

1. Domain: \((-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, \infty)\)
2. Domain: \( \{x \mid x \neq -\frac{2}{5}\} \)

We there for have a vertical asymptote \( x = -\frac{2}{5} \).

To find the range we graph the function. We will start by choosing 0, 1, -1, 2, -2 for \( x \) and solve \( f(x) \). For \( x=2 \) this is how:

\[
\begin{align*}
  f(x) &= \frac{3}{5x+2} \\
  f(2) &= \frac{3}{5(2)+2} \\
        &= \frac{3}{12}
\end{align*}
\]
Finding the other points the same way we get the following table:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{3}{2} = 1.5$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{3}{7} = 0.4$</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{3}{12} = 0.25$</td>
</tr>
<tr>
<td>-2</td>
<td>$\frac{-3}{8} = -0.4$</td>
</tr>
</tbody>
</table>

If we put everything we have so far in a picture we get:

![Graph of the function](image)

It can be helpful to choose a few more points and we will find this graph:

![Graph with additional points](image)

Knowing that we have a vertical and a horizontal asymptote we can see from here that the range is $(-\infty, 0) \cup (0, \infty)$. 
Function Transformation

We relate many functions back to a few basic function types by using transformations. That can be very helpful in graphing the function and finding its range.

<table>
<thead>
<tr>
<th>Function</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x) + k</td>
<td>Shift vertically up k units</td>
</tr>
<tr>
<td>f(x) - k</td>
<td>Shift vertically down k units</td>
</tr>
<tr>
<td>f(x+k)</td>
<td>Shift horizontally left k units</td>
</tr>
<tr>
<td>f(x-k)</td>
<td>Shift horizontally right k units</td>
</tr>
<tr>
<td>-f(x)</td>
<td>Reflect vertically about the x-axis</td>
</tr>
<tr>
<td>f(-x)</td>
<td>Reflect horizontally about the y-axis</td>
</tr>
<tr>
<td>k•f(x)</td>
<td>Stretch/Shrink vertically by a factor of k</td>
</tr>
<tr>
<td>f(k•x)</td>
<td>Stretch/Shrink horizontally by a factor of 1/k</td>
</tr>
</tbody>
</table>

Example:
The function \( f(x) = (x-2)^2 - 3 \) has the function \( g(x) = x^2 \) as the base. Looking at the above transformation table we can see that we can find the graph of \( f(x) \) by shifting \( g(x) \) 3 units down and 2 units to the right. If we know that \( g(x) \) has a range of \([0, \infty)\) we know that the range of \( f(x) \) is \([-3, \infty)\). And we can graph \( f(x) \) easily:

![Graph of function](image)

Practice Problems:
For each of the functions:
   a) Find the domain.  b) Find the range.  c) Graph the function.

1. \( f(x) = 2x^2 - 4 \)
2. \( f(x) = \frac{1}{x+2} \)
3. \( f(x) = -4x^4 - x^3 + 5x^2 - x + 1 \)
4. \( f(x) = \sqrt{2x+6} \)
5. \( f(x) = \sqrt{-x+4} - 2 \)
6. \( f(x) = \frac{x}{x^2 - x - 2} \)
Answers to Practice Problems:

1. a) all real numbers
   b) $[-4,\infty)$
   c) 

   ![Graph](image1)

2. a) $(-\infty,-2) \cup (-2,\infty)$
   b) $(-\infty,0) \cup (0,\infty)$
   c) 

   ![Graph](image2)

3. a) all real numbers
   b) $(-\infty,4)$
   c) 

   ![Graph](image3)

4. a) $[-3,\infty)$
   b) $[0,\infty)$
   c) 

   ![Graph](image4)

5. a) $(\infty,4]$ 
   b) $[-2,\infty)$
   c) 

   ![Graph](image5)

6. a) $(-\infty,-1) \cup (-1,\infty) \cup (2,\infty)$
   b) all real numbers
   c) 

   ![Graph](image6)
Laws of Logarithms
(Math 111)

The logarithm is defined as the inverse of the exponent. If we want to solve \( b^x = m \) for \( x \) then the logarithm is defined as the solution:

\[ x = \log_b m \]

When simplifying expressions with logarithms we follow the laws of logarithms, just like we did with the laws of exponents. It is important to notice that the laws of logarithms are different, inverse to be exact. That is because the logarithm is the inverse function of the exponent.

\[
\begin{align*}
\log_b m + \log_b n &= \log_b (m \cdot n) \\
\log_b m - \log_b n &= \log_b \left( \frac{m}{n} \right) \\
\log_b r^m &= m \log_b r \\
\log_b 1 &= 0 \\
\log_b b &= 1 \\
\log_b m &= \frac{\log_a m}{\log_a b}
\end{align*}
\]

The logarithms to the base 10 and e have special expressions:

\[
\begin{align*}
\log_{10} m &= \log_{10} m \\
\ln x &= \log_e x
\end{align*}
\]

Practice Problems:
Simplify, using the laws of logarithms.

1. \( \log 10 + \log 3 \)
2. \( \log 28 - \log 4 \)
3. \( 4 \ln 3 \)
4. \( \log 9 - 2 \log 3 \)
5. \( \log_{20} 4 + \log_{29} 5 \)
6. \( \ln 20 + \ln 3 - \ln 6 \)

Answers to Practice Problems:

1. \( \log 30 \)
2. \( \log 7 \)
3. \( \ln 81 \)
4. \( 0 \)
5. \( 1 \)
6. \( \ln 10 \)
Additional Math Review Resources

Online
- [www.college-cram.com](http://www.college-cram.com) Click on “Choose a Subject to Study!” and choose “Algebra”, “Pre-Calculus”, or “Trigonometry”.
- [www.math.com](http://www.math.com)
- [www.purplemath.com](http://www.purplemath.com)
- [http://mathforum.org/mathtools/sitemap.html](http://mathforum.org/mathtools/sitemap.html) This site leads to a big selection of software, some of which is free; some is not.

Learning Centers
Please note that opening hours during the summer can be different than the hours given below. Please check with the learning center.

- Cascade:  Learning Center
  TH 123, 503-978-5263, opening hours:
  - Monday to Thursday: 8am - 6pm
  - Friday: 8am - 2pm
  - Saturday & Sunday: 10am - 2pm

- Rock Creek:  Student Learning Center
  Bldg. 2, Rm. 212, 503-614-7414, opening hours:
  - Monday to Thursday: 8am - 8pm
  - Friday: 8am - 3pm
  - Saturday: 11am - 3pm

- Southeast Center:  Tutoring Center
  Mt Tabor Hall, Room 123, 503-788-6159, opening hours:
  - Monday to Thursday: 9am - 7pm
  - Friday: 9am - 3pm

- Sylvania:  Student Success Center
  CC 204, 503-977-4540, opening hours:
  - Monday to Thursday: 9am - 8pm
  - Friday: 9am - 3pm
  - Saturday: 10am - 3pm

- Tutoring website: [http://www.pcc.edu/resources/tutoring/](http://www.pcc.edu/resources/tutoring/)